SEASONAL TIME SERIES

1. SEASONAL MA(q)

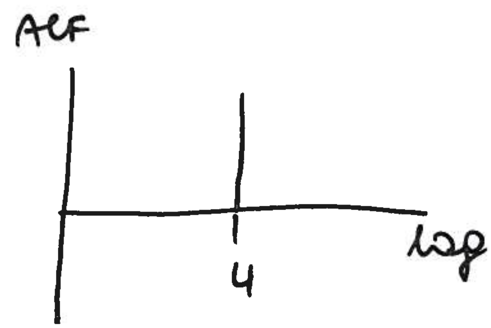
$$Z_t = \Theta_q(B^s)\,a_t$$

where $\Theta_q = 1 - \Theta_1 B^s - \Theta_2 B^{2s} \cdots - \Theta_q B^{q\cdot s}$

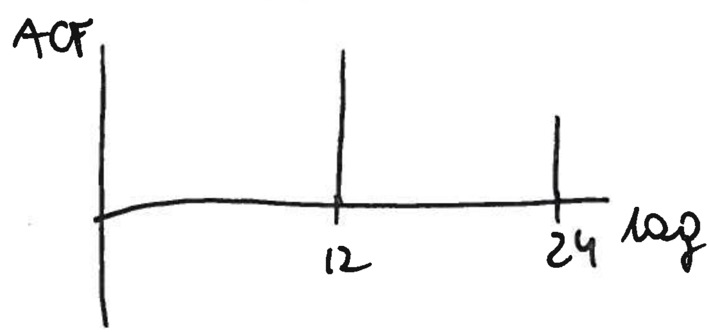
and $s=12$ for monthly patterns, or $s=4$ for quarterly

$$Z_t = a_t - \Theta_1 a_{t-s} - \Theta_2 a_{t-2s} - \cdots - \Theta_q a_{t-qs}$$

ex: $MA(1)_4$: $Z_t = a_t - \Theta_1 a_{t-4}$



ex: $MA(2)_{12}$: $Z_t = a_t - \Theta_1 a_{t-12} - \Theta_2 a_{t-24}$



AUTOCORRELATIONS $\rho_s, \rho_{2s} \ldots \rho_{qs} \neq 0$ **AND CUT OFF AT LAG qs**

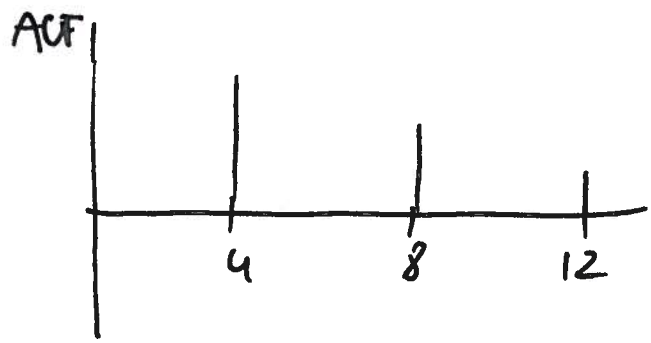
2. SEASONAL AR(p):

$$\Phi_p(B^s)\, Z_t = a_t$$

where $\Phi_p = 1 - \phi_1 B^s - \phi_2 B^{2s} - \dots - \phi_p B^{p\cdot s}$

$$Z_t = \phi_1 Z_{t-s} + \phi_2 Z_{t-2s} + \dots + \phi_p Z_{t-p\cdot s} + a_t$$

ex: $AR(1)_4$: $Z_t = \phi_1 Z_{t-4} + a_t$



AUTOCORRELATIONS $\rho_{js} = \phi_1^{\,j}$ AND DIE OUT EXPONEN

ALL OTHER autocorrelations are 0.

- IF YOU OBSERVE THAT THE AUTOCORRELATIONS AT SEASONAL LAGS DIE OUT VERY SLOWLY, AT A LINEAR RATE, TRANSFORM Z_t into $(1-B^s)Z_t = Z_t - Z_{t-s}$

- IF THE DATA DISPLAY BOTH TREND AND AUTOCORRALTION AT SEASONAL LAGS THAT DIE OUT AT A LINEAR RATE TRANSFORM z_t INTO $(1-B)(1-B^S)\,z_t \;:\Rightarrow \Delta\,\Delta_S\, z_t$

z_t:	Δz_t	$(1-B^S)\,\Delta z_t$ (ex $S=4$)
z_1	•	•
z_2	$\Delta z_2 = z_2 - z_1$	•
z_3	$\Delta z_3 = z_3 - z_2$	•
z_4	$\Delta z_4 = z_4 - z_3$	•
z_5	$\Delta z_5 = z_5 - z_4$	•
z_6	$\Delta z_6 = z_6 - z_5$	$\Delta z_6 - \Delta z_2$
z_7	$\Delta z_7 = z_7 - z_6$	$\Delta z_7 - \Delta z_3$
z_8	$\Delta z_8 = z_8 - z_7$	$\Delta z_8 - \Delta z_4$

SEASONALITY IS A PREDICTABLE PATTERN THAT NEEDS TO BE REMOVED, OR INCLUDED IN THE MODEL:

$$\Phi_{p_1}(B^S)\cdot \Phi_{p_2}(B)\, z_t = \Theta_{q_1}(B^S)\, \Theta_{q_2}(B)\, a_t$$

IF THAT IS TOO DIFFICULT, USE DESEASONALIZED DATA OR APPLY FILTER X11 (RATS, SAS)

INTEGRATED ARMA(p,q)

- IS CALLED ARIMA(p,d,q) where d is the order of INTEGRATION:

$$(1-B)^{d}\,\Phi_p(B)\,z_t = \Theta_q(B)\,a_t$$

IT IS A NONSTATIONARY PROCESS. IN MOST CASES $d=1$, and the process is "integrated of order one".
THAT PROCESS NEEDS TO BE TRANSFORMED INTO FIRST DIFFERENCES PRIOR TO THE MODELLING:

$$(1-B)\,z_t = \Delta z_t = z_t - z_{t-1}$$

Next, we can estimate:

$$\Phi_p(B)\,\Delta z_t = \Theta_q(B)\,a_t$$

USING THE STANDARD METHODS:

- THE AUTOREGRESSIVE PROCESS, INTEGRATED OF ORDER 1 IS CALLED THE **RANDOM WALK:**

$$z_t = z_{t-1} + a_t$$

ITS FIRST DIFFERENCE $\Delta z_t = a_t$ IS STATIONARY