ADDITIONAL RESULTS on CAUCHY and N(0,1) Mixtures for Misspecification of Causal and Noncausal Orders in Autoregressive Processes

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Abstract

These tables contain simulation results that clarify the consequences of estimating a (past-dependent) causal AR model from data generated by a stationary (future-dependent) noncausal process. The data are a mixture of a Cauchy distribution and N(0,1).

Keywords: Noncausal Process, Misspecification, Binding Function, Indirect Inference.

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0.1 Introduction

The data are generated from the mixture of standard Cauchy and standard Normal distributions. The ratio of normal variabls in the sample, denoted by λ, is assumed to be known. We examine the behavior of the autoregressive coefficient estimator $\hat{\rho}(\lambda_0, \rho_0)$ as a function of λ_0, and the persistence parameter ρ_0 .

We use 1000 replications of noncausal AR(1) trajectories of length $T = 400$ and $T = 100$, which are referred to as the large and the small sample, respectively. The true parameter ρ_0 takes the values 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9. Parameter λ_0 is set equal to the following values: 0.0, 0.01, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.85, 0.9, 0.95 and 1.

Tables 1.0 to 1.9 display the behavior of the PML estimators of $\hat{\rho}(\lambda_0, \rho_0)$, called the Mix PMLE and Cauchy PMLE and given in columns 1 and 3, respectively, and compare them to the OLS-based first-order autocorrelation estimator, denoted by OLS and given in column 2.

For each estimator, we report the median and the 5th and 95th percentiles of its sampling distribution.

The OLS estimators lead to a misleading conclusion that the model is causal while it is not.

The PML estimators underestimate the autoregressive coefficient when it is below 0.5 and overestimate it otherwise.

Table 1.0: Estimates of $\rho = 0.0$

lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
	T=400								
	Mix PMLE			**OLS**			**Cauchy PMLE**		
0.0	0.000	-0.0093	0.0108	-0.0024	-0.0548	0.0444	0.0000	-0.0093	0.0108
0.0100	0.0001	-0.0094	0.0091	-0.0024	-0.0541	0.0431	0.0000	-0.0097	0.0103
0.1000	0.0001	-0.0109	0.0100	-0.0024	-0.0451	0.0415	0.0001	-0.0104	0.0098
0.2000	0.0000	-0.0115	0.0114	-0.0024	-0.0486	0.0408	0.0000	-0.0105	0.0113
0.3000	0.0001	-0.0113	0.0107	-0.0023	-0.0475	0.0437	0.0000	-0.0104	0.0124
0.4000	0.0000	-0.0127	0.0113	-0.0024	-0.0489	0.0399	0.0000	-0.0130	0.0148
0.5000	0.0000	-0.0150	0.0118	-0.0024	-0.0509	0.0448	0.0001	-0.0135	0.0126
0.6000	0.0001	-0.0179	0.0174	-0.0024	-0.0493	0.0367	0.0002	-0.0167	0.0188
0.7000	0.0000	-0.0206	0.0175	-0.0027	-0.0510	0.0421	0.0000	-0.0192	0.0238
0.8000	-0.0002	-0.0266	0.0272	-0.0020	-0.0532	0.0405	0.0000	-0.0288	0.0277
0.8500	0.0000	-0.0330	0.0276	-0.0026	-0.0574	0.0470	0.0001	-0.0373	0.0413
0.9000	0.0000	-0.0406	0.0422	-0.0033	-0.0606	0.0558	0.0003	-0.0507	0.0486
0.9500	-0.0001	-0.0638	0.0543	-0.0028	-0.0742	0.0609	0.0002	-0.0647	0.0690
1.0000	-0.0081	-0.0821	0.0749	-0.0081	-0.0821	0.0749	-0.0006	-0.0970	0.1007
lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
	T=100								
	Mix PMLE			**OLS**			**Cauchy PMLE**		
0.0000	0.0003	-0.0334	0.0382	-0.0104	-0.1246	0.1105	0.0003	-0.0334	0.0382
0.0100	0.0000	-0.0347	0.0395	-0.0103	-0.1219	0.1171	0.0003	-0.0364	0.0423
0.1000	0.0001	-0.0395	0.0443	-0.0102	-0.1191	0.0970	0.0000	-0.0439	0.0441
0.2000	0.0000	-0.0483	0.0452	-0.0108	-0.1335	0.1008	0.0001	-0.0441	0.0401
0.3000	-0.0001	-0.0437	0.0485	-0.0094	-0.1161	0.1067	0.0002	-0.0432	0.0517
0.4000	0.0001	-0.0454	0.0536	-0.0107	-0.1149	0.1020	-0.0003	-0.0456	0.0518
0.5000	0.0004	-0.0474	0.0558	-0.0109	-0.1177	0.1171	0.0001	-0.0583	0.0598
0.6000	0.0002	-0.0585	0.0617	-0.0105	-0.1256	0.0903	0.0000	-0.0715	0.0729
0.7000	-0.0006	-0.0748	0.0755	-0.0113	-0.1275	0.0947	-0.0001	-0.0875	0.0780
0.8000	-0.0001	-0.0898	0.0840	-0.0115	-0.1328	0.1239	-0.0005	-0.1097	0.1014
0.8500	-0.0023	-0.1102	0.0969	-0.0124	-0.1394	0.1176	0.0002	-0.1171	0.1100
0.9000	-0.0004	-0.1317	0.1097	-0.0110	-0.1535	0.1357	-0.0013	-0.1364	0.1281
0.9500	-0.0028	-0.1481	0.1388	-0.0113	-0.1645	0.1391	-0.0045	-0.1750	0.1559
1.0000	-0.0105	-0.1700	0.1622	-0.0105	-0.1700	0.1622	-0.0050	-0.1964	0.1879

Table 1.1: Estimates of $\rho = 0.1$

lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
	T=400								
	Mix PMLE			**OLS**			**Cauchy PMLE**		
0.0000	0.0002	-0.0085	0.0158	0.0976	0.0551	0.1466	0.0002	-0.0085	0.0158
0.0100	0.0001	-0.0082	0.0141	0.0976	0.0549	0.1445	0.0001	-0.0082	0.0136
0.1000	0.0002	-0.0080	0.0138	0.0976	0.0595	0.1454	0.0002	-0.0081	0.0147
0.2000	0.0002	-0.0087	0.0173	0.0978	0.0617	0.1384	0.0002	-0.0078	0.0149
0.3000	0.0003	-0.0084	0.0161	0.0974	0.0549	0.1415	0.0003	-0.0080	0.0181
0.4000	0.0002	-0.0103	0.0199	0.0976	0.0608	0.1402	0.0004	-0.0089	0.0195
0.5000	0.0006	-0.0112	0.0227	0.0975	0.0585	0.1406	0.0006	-0.0104	0.0198
0.6000	0.0010	-0.0132	0.0295	0.0973	0.0582	0.1426	0.0008	-0.0116	0.0318
0.7000	0.0013	-0.0129	0.0385	0.0969	0.0504	0.1381	0.0013	-0.0133	0.0438
0.8000	0.0037	-0.0119	0.0582	0.0973	0.0513	0.1390	0.0033	-0.0145	0.0618
0.8500	0.0078	-0.0110	0.0793	0.0973	0.0447	0.1496	0.0059	-0.0130	0.0812
0.9000	0.0155	-0.0109	0.1076	0.0967	0.0356	0.1537	0.0107	-0.0166	0.1094
0.9500	0.0406	-0.0103	0.1400	0.0968	0.0252	0.1665	0.0334	-0.0168	0.1420
1.0000	0.0981	0.0190	0.1856	0.0981	0.0190	0.1856	0.0944	0.0129	0.1941
lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
	T=100								
	Mix PMLE			**OLS**			**Cauchy PMLE**		
0.0000	0.0014	-0.0396	0.0607	0.0887	-0.0214	0.1790	0.0014	-0.0396	0.0607
0.0100	0.0014	-0.0346	0.0582	0.0888	-0.0213	0.1816	0.0005	-0.0391	0.0513
0.1000	0.0016	-0.0354	0.0614	0.0886	-0.0196	0.1812	0.0008	-0.0402	0.0532
0.2000	0.0016	-0.0359	0.0660	0.0889	-0.0254	0.1928	0.0009	-0.0406	0.0512
0.3000	0.0026	-0.0368	0.0632	0.0889	-0.0148	0.1936	0.0009	-0.0376	0.0671
0.4000	0.0014	-0.0384	0.0777	0.0896	-0.0245	0.1910	0.0010	-0.0364	0.0800
0.5000	0.0022	-0.0452	0.0751	0.0894	-0.0183	0.1922	0.0020	-0.0482	0.0778
0.6000	0.0035	-0.0435	0.0937	0.0884	-0.0235	0.2131	0.0034	-0.0512	0.1065
0.7000	0.0066	-0.0437	0.1229	0.0889	-0.0391	0.2069	0.0048	-0.0523	0.1247
0.8000	0.0132	-0.0511	0.1570	0.0886	-0.0444	0.2157	0.0104	-0.0601	0.1720
0.8500	0.0222	-0.0530	0.1845	0.0871	-0.0694	0.2201	0.0172	-0.0659	0.1975
0.9000	0.0297	-0.0627	0.2051	0.0886	-0.0459	0.2358	0.0290	-0.0728	0.2243
0.9500	0.0571	-0.0667	0.2495	0.0887	-0.0640	0.2268	0.0612	-0.0794	0.2648
1.0000	0.0922	-0.0741	0.2729	0.0922	-0.0741	0.2729	0.1068	-0.0906	0.2892

Table 1.2: Estimates of $\rho = 0.2$

lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
	T=400								
	Mix PMLE			**OLS**			**Cauchy PMLE**		
0.0000	0.0006	-0.0076	0.0255	0.1968	0.1548	0.2422	0.0006	-0.0076	0.0255
0.0100	0.0007	-0.0073	0.0253	0.1970	0.1553	0.2422	0.0007	-0.0073	0.0207
0.1000	0.0010	-0.0070	0.0283	0.1969	0.1520	0.2369	0.0008	-0.0084	0.0218
0.2000	0.0009	-0.0068	0.0266	0.1969	0.1559	0.2448	0.0007	-0.0078	0.0248
0.3000	0.0012	-0.0070	0.0323	0.1972	0.1560	0.2367	0.0012	-0.0072	0.0309
0.4000	0.0013	-0.0081	0.0384	0.1970	0.1517	0.2485	0.0012	-0.0079	0.0332
0.5000	0.0023	-0.0067	0.0399	0.1969	0.1567	0.2430	0.0019	-0.0080	0.0414
0.6000	0.0035	-0.0069	0.0485	0.1968	0.1506	0.2358	0.0027	-0.0069	0.0558
0.7000	0.0053	-0.0063	0.0770	0.1969	0.1599	0.2412	0.0048	-0.0072	0.0779
0.8000	0.0168	-0.0051	0.1272	0.1974	0.1538	0.2543	0.0112	-0.0083	0.1237
0.8500	0.0332	-0.0033	0.1623	0.1970	0.1519	0.2512	0.0210	-0.0086	0.1651
0.9000	0.0615	-0.0013	0.1935	0.1980	0.1382	0.2575	0.0646	-0.0042	0.2106
0.9500	0.1363	0.0069	0.2405	0.1975	0.1351	0.2669	0.1367	-0.0001	0.2449
1.0000	0.2011	0.1189	0.2803	0.2011	0.1189	0.2803	0.2037	0.1118	0.2878
lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
	T=100								
	Mix PMLE			**OLS**			**Cauchy PMLE**		
0.0000	0.0035	-0.0273	0.0992	0.1884	0.0581	0.3035	0.0035	-0.0273	0.0992
0.0100	0.0026	-0.0253	0.0989	0.1884	0.0596	0.3022	0.0034	-0.0267	0.0863
0.1000	0.0031	-0.0230	0.1030	0.1873	0.0616	0.3059	0.0029	-0.0285	0.1037
0.2000	0.0036	-0.0274	0.1112	0.1875	0.0770	0.3177	0.0046	-0.0262	0.1006
0.3000	0.0054	-0.0252	0.1190	0.1877	0.0666	0.3108	0.0038	-0.0266	0.1088
0.4000	0.0072	-0.0241	0.1338	0.1879	0.0751	0.3104	0.0068	-0.0251	0.1225
0.5000	0.0078	-0.0305	0.1397	0.1869	0.0692	0.3166	0.0074	-0.0289	0.1365
0.6000	0.0138	-0.0280	0.1654	0.1877	0.0666	0.2944	0.0091	-0.0306	0.1498
0.7000	0.0182	-0.0219	0.2196	0.1884	0.0650	0.3217	0.0140	-0.0325	0.2170
0.8000	0.0367	-0.0205	0.2508	0.1869	0.0624	0.3105	0.0298	-0.0286	0.2688
0.8500	0.0625	-0.0226	0.2737	0.1880	0.0516	0.3036	0.0528	-0.0318	0.2826
0.9000	0.0961	-0.0141	0.2825	0.1869	0.0340	0.3287	0.0904	-0.0291	0.3136
0.9500	0.1374	-0.0128	0.3126	0.1885	0.0181	0.3378	0.1379	-0.0238	0.3412
1.0000	0.1943	0.0388	0.3539	0.1943	0.0388	0.3539	0.1947	0.0071	0.3803

Table 1.3: Estimates of $\rho = 0.3$

lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
	T=400								
	Mix PMLE			**OLS**			**Cauchy PMLE**		
0.0000	0.0022	-0.0048	0.0460	0.2970	0.2471	0.3428	0.0022	-0.0048	0.0460
0.0100	0.0020	-0.0053	0.0474	0.2968	0.2482	0.3419	0.0023	-0.0048	0.0437
0.1000	0.0018	-0.0060	0.0521	0.2968	0.2474	0.3448	0.0023	-0.0052	0.0510
0.2000	0.0027	-0.0051	0.0531	0.2968	0.2494	0.3398	0.0023	-0.0052	0.0526
0.3000	0.0036	-0.0052	0.0596	0.2971	0.2488	0.3426	0.0027	-0.0044	0.0657
0.4000	0.0047	-0.0043	0.0724	0.2967	0.2548	0.3423	0.0030	-0.0049	0.0733
0.5000	0.0062	-0.0049	0.0867	0.2966	0.2433	0.3425	0.0051	-0.0054	0.0882
0.6000	0.0097	-0.0034	0.1183	0.2962	0.2468	0.3393	0.0073	-0.0049	0.1161
0.7000	0.0213	-0.0026	0.1801	0.2968	0.2464	0.3400	0.0165	-0.0045	0.1854
0.8000	0.0703	-0.0007	0.2409	0.2966	0.2487	0.3381	0.0551	-0.0029	0.2446
0.8500	0.1101	0.0000	0.2738	0.2969	0.2476	0.3456	0.1297	-0.0005	0.2859
0.9000	0.1705	0.0043	0.3116	0.2970	0.2363	0.3512	0.1989	0.0006	0.3206
0.9500	0.2376	0.0659	0.3462	0.2963	0.2323	0.3581	0.2519	0.0876	0.3589
1.0000	0.2999	0.2156	0.3746	0.2999	0.2156	0.3746	0.3009	0.2057	0.3879
lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
	T=100								
	Mix PMLE			**OLS**			**Cauchy PMLE**		
0.0000	0.0075	-0.0277	0.2065	0.2858	0.1564	0.4038	0.0075	-0.0277	0.2065
0.0100	0.0071	-0.0287	0.1928	0.2857	0.1628	0.4035	0.0075	-0.0206	0.1940
0.1000	0.0059	-0.0296	0.2132	0.2857	0.1563	0.4077	0.0084	-0.0208	0.2002
0.2000	0.0095	-0.0244	0.1930	0.2859	0.1774	0.3971	0.0091	-0.0253	0.2075
0.3000	0.0124	-0.0224	0.2205	0.2852	0.1554	0.4001	0.0104	-0.0216	0.1904
0.4000	0.0151	-0.0222	0.2304	0.2864	0.1679	0.4009	0.0120	-0.0229	0.2206
0.5000	0.0229	-0.0192	0.2591	0.2849	0.1473	0.4054	0.0151	-0.0241	0.2570
0.6000	0.0342	-0.0184	0.2932	0.2855	0.1583	0.4089	0.0238	-0.0185	0.3185
0.7000	0.0582	-0.0133	0.3258	0.2858	0.1595	0.4087	0.0380	-0.0184	0.3365
0.8000	0.0994	-0.0112	0.3770	0.2865	0.1581	0.4084	0.1067	-0.0141	0.3833
0.8500	0.1531	-0.0050	0.3891	0.2860	0.1296	0.4215	0.1387	-0.0114	0.3853
0.9000	0.2039	-0.0001	0.4168	0.2848	0.1408	0.4263	0.1930	-0.0047	0.4150
0.9500	0.2574	0.0200	0.4261	0.2867	0.1302	0.4268	0.2481	-0.0033	0.4470
1.0000	0.3011	0.1268	0.4319	0.3011	0.1268	0.4319	0.3028	0.0925	0.4598

Table 1.4: Estimates of $\rho = 0.4$

lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
	T=400								
	Mix PMLE			**OLS**			**Cauchy PMLE**		
0.0000	0.0103	-0.0034	0.1960	0.3964	0.3546	0.4440	0.0103	-0.0034	0.1960
0.0100	0.0076	-0.0037	0.1753	0.3964	0.3545	0.4430	0.0101	-0.0029	0.1991
0.1000	0.0079	-0.0037	0.1814	0.3965	0.3530	0.4505	0.0098	-0.0031	0.2012
0.2000	0.0109	-0.0032	0.1972	0.3963	0.3503	0.4420	0.0129	-0.0031	0.2167
0.3000	0.0124	-0.0026	0.2159	0.3967	0.3540	0.4463	0.0127	-0.0030	0.2356
0.4000	0.0177	-0.0024	0.2431	0.3961	0.3508	0.4404	0.0187	-0.0020	0.2505
0.5000	0.0308	-0.0022	0.3206	0.3964	0.3558	0.4400	0.0329	-0.0025	0.2863
0.6000	0.0686	-0.0016	0.3426	0.3965	0.3517	0.4411	0.0732	-0.0014	0.3306
0.7000	0.1582	0.0004	0.4011	0.3965	0.3490	0.4443	0.1658	-0.0005	0.3818
0.8000	0.2631	0.0060	0.4225	0.3961	0.3491	0.4377	0.2666	0.0054	0.4331
0.8500	0.3041	0.0418	0.4400	0.3959	0.3346	0.4452	0.3118	0.0361	0.4429
0.9000	0.3349	0.1404	0.4555	0.3959	0.3408	0.4530	0.3422	0.1727	0.4517
0.9500	0.3724	0.2357	0.4656	0.3965	0.3311	0.4576	0.3708	0.2582	0.4719
1.0000	0.4000	0.3233	0.4712	0.4000	0.3233	0.4712	0.3974	0.3066	0.4841
lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
	T=100								
	Mix PMLE			**OLS**			**Cauchy PMLE**		
0.0000	0.0547	-0.0166	0.6310	0.3848	0.2676	0.4982	0.0547	-0.0166	0.6310
0.0100	0.0508	-0.0166	0.6466	0.3850	0.2708	0.4999	0.0387	-0.0157	0.4214
0.1000	0.0528	-0.0153	0.6347	0.3853	0.2783	0.4991	0.0408	-0.0149	0.4053
0.2000	0.0478	-0.0141	0.6429	0.3857	0.2620	0.5029	0.0419	-0.0150	0.4122
0.3000	0.0599	-0.0121	0.5970	0.3851	0.2615	0.4903	0.0542	-0.0144	0.4334
0.4000	0.0764	-0.0104	0.5871	0.3843	0.2781	0.5000	0.0582	-0.0103	0.4653
0.5000	0.0978	-0.0073	0.6013	0.3861	0.2707	0.4949	0.0922	-0.0105	0.5139
0.6000	0.1334	-0.0078	0.5571	0.3854	0.2557	0.4982	0.1094	-0.0088	0.5104
0.7000	0.2051	-0.0030	0.5859	0.3861	0.2623	0.5169	0.1890	-0.0048	0.5480
0.8000	0.2865	0.0006	0.5541	0.3838	0.2522	0.4953	0.2877	-0.0018	0.5569
0.8500	0.3107	0.0268	0.5376	0.3830	0.2517	0.5147	0.3060	0.0005	0.5579
0.9000	0.3354	0.0458	0.5473	0.3846	0.2400	0.5098	0.3422	0.0248	0.5574
0.9500	0.3778	0.1333	0.5465	0.3835	0.2283	0.5222	0.3695	0.0925	0.5634
1.0000	0.4056	0.2384	0.5370	0.4056	0.2384	0.5370	0.3940	0.2172	0.5686

Table 1.5: Estimates of $\rho = 0.5$

lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
	T=400								
	Mix PMLE			**OLS**			**Cauchy PMLE**		
0.0000	0.2895	0.0031	1.9889	0.4961	0.4462	0.5354	0.2895	0.0031	1.9889
0.0100	0.3943	0.0123	1.9881	0.4961	0.4450	0.5349	0.2865	0.0029	1.9380
0.1000	0.3773	0.0105	1.9615	0.4963	0.4409	0.5357	0.2966	0.0015	1.8270
0.2000	0.5018	0.0256	1.9547	0.4961	0.4507	0.5392	0.3123	0.0020	1.5902
0.3000	0.5017	0.0235	1.6148	0.4961	0.4390	0.5336	0.3485	0.0049	1.4594
0.4000	0.4994	0.0288	1.2169	0.4961	0.4372	0.5390	0.3037	0.0100	1.1022
0.5000	0.5125	0.0771	0.9732	0.4963	0.4390	0.5435	0.4464	0.0204	0.9453
0.6000	0.5026	0.3095	0.6560	0.4961	0.4407	0.5477	0.4836	0.0979	0.8061
0.7000	0.4989	0.3359	0.6320	0.4962	0.4392	0.5388	0.4998	0.2413	0.7395
0.8000	0.5058	0.3676	0.6166	0.4957	0.4419	0.5428	0.5035	0.3269	0.6476
0.8500	0.4979	0.3929	0.5914	0.4958	0.4413	0.5456	0.4994	0.3579	0.6284
0.9000	0.4965	0.4190	0.5665	0.4961	0.4386	0.5455	0.5023	0.3777	0.6058
0.9500	0.5026	0.3975	0.5884	0.4957	0.4328	0.5554	0.4985	0.4050	0.5969
1.0000	0.4985	0.4236	0.5670	0.4985	0.4236	0.5670	0.4975	0.4109	0.5775
lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
	T=100								
	Mix PMLE			**OLS**			**Cauchy PMLE**		
0.0000	0.2956	-0.0003	1.9983	0.4839	0.3609	0.6034	0.2956	-0.003	1.9983
0.0100	0.3059	0.0015	1.9960	0.4839	0.3578	0.6034	0.2954	-0.0003	1.8226
0.1000	0.4208	0.0032	1.9918	0.4841	0.3698	0.6082	0.3000	0.0001	1.7127
0.2000	0.5146	0.0058	1.9966	0.4837	0.3746	0.6125	0.3265	0.0019	1.7050
0.3000	0.5117	0.0038	1.9790	0.4831	0.3535	0.5936	0.3284	-0.0009	1.6713
0.4000	0.5154	0.0067	1.8983	0.4849	0.3693	0.6127	0.3709	0.0011	1.7174
0.5000	0.5093	0.0092	1.7158	0.4833	0.3396	0.6074	0.4296	0.0062	1.4924
0.6000	0.4966	0.1335	0.8097	0.4824	0.3506	0.6004	0.4704	0.0284	1.1954
0.7000	0.5032	0.1477	0.7998	0.4833	0.3576	0.5934	0.4918	0.0178	0.9573
0.8000	0.4964	0.2305	0.7158	0.4826	0.3521	0.6043	0.5030	0.0526	0.8762
0.8500	0.4947	0.2656	0.6844	0.4832	0.3484	0.6081	0.5079	0.1290	0.8049
0.9000	0.4947	0.3360	0.6214	0.4821	0.3392	0.6142	0.5002	0.2263	0.7051
0.9500	0.4949	0.2980	0.6731	0.4874	0.3451	0.6141	0.4999	0.2872	0.6692
1.0000	0.4948	0.3425	0.6283	0.4948	0.3425	0.6283	0.4977	0.3300	0.6393

Table 1.6: Estimates of $\rho = 0.6$

lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
	T=400								
	Mix PMLE			**OLS**			**Cauchy PMLE**		
0.0000	1.6499	1.2725	1.6683	0.5960	0.5518	0.6357	1.6499	1.2725	1.6683
0.0100	1.6492	1.2520	1.6683	0.5960	0.5526	0.6358	1.6446	1.2117	1.6684
0.1000	1.6448	1.2097	1.6683	0.5960	0.5515	0.6355	1.6381	1.1755	1.6680
0.2000	1.6341	1.1402	1.6678	0.5961	0.5483	0.6359	1.6237	1.0886	1.6680
0.3000	1.6135	1.0352	1.6678	0.5957	0.5512	0.6303	1.6043	1.0146	1.6679
0.4000	1.5343	0.9012	1.6673	0.5960	0.5507	0.6384	1.5509	0.8870	1.6672
0.5000	1.3090	0.7904	1.6668	0.5959	0.5552	0.6373	1.3377	0.7894	1.6670
0.6000	1.0809	0.7255	1.6658	0.5956	0.5504	0.6420	1.0954	0.7127	1.6664
0.7000	0.8901	0.6443	1.6310	0.5959	0.5480	0.6411	0.8851	0.6562	1.6571
0.8000	0.7601	0.5996	1.0608	0.5960	0.5473	0.6414	0.7550	0.6057	1.0778
0.8500	0.7177	0.5829	0.9301	0.5954	0.5421	0.6371	0.7098	0.5740	0.8752
0.9000	0.6741	0.5598	0.8284	0.5957	0.5425	0.6415	0.6705	0.5521	0.7874
0.9500	0.6383	0.5474	0.7403	0.5956	0.5392	0.6529	0.6323	0.5377	0.7310
1.0000	0.6002	0.5294	0.6610	0.6002	0.5294	0.6610	0.6024	0.5232	0.6728
lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
	T=100								
	Mix PMLE			**OLS**			**Cauchy PMLE**		
0.0000	1.5358	0.5173	1.6721	0.5845	0.4660	0.7089	1.5358	0.5173	1.6721
0.0100	1.5338	0.5163	1.6719	0.5845	0.4652	0.7103	1.4576	0.2092	1.6719
0.1000	1.5283	0.5670	1.6720	0.5842	0.4715	0.7089	1.4421	0.2129	1.6700
0.2000	1.4749	0.5172	1.6714	0.5836	0.4681	0.7130	1.4200	0.2360	1.6709
0.3000	1.4432	0.4926	1.6710	0.5831	0.4697	0.7076	1.3262	0.2240	1.6703
0.4000	1.3350	0.5239	1.6703	0.5835	0.4554	0.7003	1.2683	0.2952	1.6697
0.5000	1.1840	0.4969	1.6674	0.5829	0.4668	0.6930	1.1276	0.3024	1.6674
0.6000	0.9855	0.4891	1.6663	0.5827	0.4650	0.6910	0.9789	0.2868	1.6673
0.7000	0.8526	0.4625	1.6600	0.5842	0.4305	0.7072	0.8419	0.4115	1.6661
0.8000	0.7469	0.4491	1.5501	0.5836	0.4642	0.6948	0.7293	0.4243	1.6441
0.8500	0.6982	0.4521	1.2485	0.5824	0.4459	0.7060	0.6930	0.4206	1.4897
0.9000	0.6620	0.4248	1.0383	0.5832	0.4358	0.6978	0.6539	0.4254	1.0218
0.9500	0.6169	0.4298	0.8352	0.5818	0.4353	0.6957	0.6195	0.4219	0.8564
1.0000	0.5886	0.4427	0.7097	0.5886	0.4427	0.7097	0.5960	0.4156	0.7365

Table 1.7: Estimates of $\rho = 0.7$

lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
	T=400								
	Mix PMLE			**OLS**			**Cauchy PMLE**		
0.0000	1.4233	1.3542	1.4302	0.6957	0.6550	0.7410	1.4233	1.3542	1.4302
0.0100	1.4231	1.3497	1.4304	0.6957	0.6555	0.7408	1.4234	1.3558	1.4301
0.1000	1.4224	1.3423	1.4305	0.6957	0.6523	0.7393	1.4229	1.3380	1.4301
0.2000	1.4213	1.3304	1.4300	0.6951	0.6455	0.7395	1.4208	1.3042	1.4301
0.3000	1.4181	1.2844	1.4299	0.6955	0.6529	0.7443	1.4193	1.2841	1.4297
0.4000	1.4160	1.2637	1.4298	0.6957	0.6512	0.7395	1.4158	1.2529	1.4299
0.5000	1.4094	1.1809	1.4293	0.6956	0.6538	0.7403	1.4091	1.1730	1.4297
0.6000	1.3936	1.0723	1.4291	0.6961	0.6536	0.7351	1.3909	1.0567	1.4297
0.7000	1.2750	0.9456	1.4287	0.6956	0.6496	0.7330	1.2943	0.9115	1.4290
0.8000	1.0490	0.8218	1.4278	0.6955	0.6499	0.7423	1.0480	0.7997	1.4284
0.8500	0.9433	0.7636	1.4214	0.6964	0.6538	0.7429	0.9244	0.7560	1.4276
0.9000	0.8667	0.7250	1.2530	0.6953	0.6493	0.7406	0.8385	0.7063	1.4056
0.9500	0.7760	0.6710	0.9571	0.6960	0.6383	0.7438	0.7612	0.6657	0.8926
1.0000	0.7001	0.6358	0.7541	0.7001	0.6358	0.7541	0.6980	0.6276	0.7573
lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
	T=100								
	Mix PMLE			**OLS**			**Cauchy PMLE**		
0.0000	1.4098	1.1366	1.4359	0.6816	0.5488	0.7832	1.4098	1.1366	1.4359
0.0100	1.4090	1.1301	1.4356	0.6816	0.5488	0.7784	1.4056	1.0637	1.4353
0.1000	1.4043	1.1076	1.4347	0.6815	0.5444	0.7798	1.4009	1.0190	1.4354
0.2000	1.4025	1.0919	1.4344	0.6820	0.5496	0.7869	1.3983	1.0160	1.4341
0.3000	1.3943	1.0294	1.4335	0.6825	0.5467	0.7810	1.3887	0.9625	1.4344
0.4000	1.3805	0.9393	1.4332	0.6805	0.5480	0.7935	1.3794	0.8982	1.4330
0.5000	1.3560	0.8344	1.4339	0.6821	0.5582	0.7875	1.3410	0.8646	1.4338
0.6000	1.2999	0.7858	1.4322	0.6800	0.5473	0.7782	1.3200	0.7778	1.4324
0.7000	1.2138	0.7373	1.4299	0.6806	0.5679	0.7822	1.1867	0.7308	1.4304
0.8000	1.0039	0.6565	1.4270	0.6800	0.5522	0.7747	0.9878	0.6319	1.4284
0.8500	0.9271	0.6318	1.4261	0.6826	0.5460	0.7872	0.8796	0.6274	1.4283
0.9000	0.8217	0.6028	1.4054	0.6797	0.5430	0.7840	0.8075	0.5801	1.4192
0.9500	0.7646	0.5738	1.3286	0.6779	0.5451	0.7852	0.7445	0.5696	1.3498
1.0000	0.6925	0.5421	0.8025	0.6767	0.5350	0.7850	0.6915	0.5353	0.8138

Table 1.8: Estimates of $\rho = 0.8$

lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
	T=400								
	Mix PMLE			**OLS**			**Cauchy PMLE**		
0.0000	1.2477	1.2249	1.2515	0.7954	0.7506	0.8347	1.2477	1.2249	1.2515
0.0100	1.2480	1.2234	1.2518	0.7955	0.7479	0.8347	1.2477	1.2245	1.2515
0.1000	1.2475	1.2201	1.2517	0.7953	0.7524	0.8302	1.2474	1.2183	1.2516
0.2000	1.2468	1.2166	1.2514	0.7953	0.7450	0.8292	1.2472	1.2184	1.2516
0.3000	1.2461	1.2079	1.2512	0.7955	0.7475	0.8293	1.2466	1.2061	1.2518
0.4000	1.2458	1.2007	1.2515	0.7953	0.7573	0.8363	1.2450	1.1935	1.2514
0.5000	1.2437	1.1810	1.2515	0.7950	0.7548	0.8317	1.2435	1.1796	1.2513
0.6000	1.2390	1.1477	1.2509	0.7954	0.7537	0.8301	1.2395	1.1501	1.2509
0.7000	1.2326	1.0805	1.2506	0.7957	0.7536	0.8334	1.2300	1.0757	1.2511
0.8000	1.2064	0.9841	1.2504	0.7952	0.7554	0.8252	1.1986	0.9690	1.2503
0.8500	1.1523	0.9112	1.2500	0.7951	0.7484	0.8365	1.1177	0.9021	1.2500
0.9000	1.0421	0.8548	1.2496	0.7951	0.7463	0.8357	1.0211	0.8394	1.2493
0.9500	0.9156	0.7925	1.2441	0.7947	0.7475	0.8390	0.8860	0.7829	1.2444
1.0000	0.7987	0.7447	0.8435	0.7940	0.7369	0.8414	0.7968	0.7351	0.8515
lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
	T=100								
	Mix PMLE			**OLS**			**Cauchy PMLE**		
0.0000	1.2423	1.1491	1.2571	0.7791	0.6641	0.8876	1.2423	1.1491	1.2571
0.0100	1.2422	1.1449	1.2571	0.7798	0.6652	0.8874	1.2422	1.1518	1.2582
0.1000	1.2410	1.1279	1.2562	0.7803	0.6601	0.8870	1.2415	1.1277	1.2577
0.2000	1.2387	1.1249	1.2570	0.7791	0.6484	0.8795	1.2394	1.1266	1.2571
0.3000	1.2346	1.1024	1.2561	0.7778	0.6589	0.8999	1.2372	1.0825	1.2568
0.4000	1.2303	1.0847	1.2550	0.7795	0.6597	0.8810	1.2325	1.0566	1.2561
0.5000	1.2217	1.0247	1.2543	0.7803	0.6709	0.8861	1.2267	1.0068	1.2574
0.6000	1.2120	0.9652	1.2532	0.7807	0.6646	0.9081	1.2115	0.9659	1.2552
0.7000	1.1775	0.8908	1.2527	0.7801	0.6733	0.9007	1.1798	0.8747	1.2545
0.8000	1.1162	0.7959	1.2510	0.7765	0.6443	0.8710	1.0914	0.8040	1.2513
0.8500	1.0443	0.7630	1.2504	0.7784	0.6501	0.8650	1.0282	0.7716	1.2513
0.9000	0.9537	0.7272	1.2471	0.7799	0.6530	0.8694	0.9514	0.7289	1.2500
0.9500	0.8500	0.6985	1.2256	0.7733	0.6519	0.8617	0.8526	0.7015	1.2295
1.0000	0.7919	0.6649	0.8802	0.7719	0.6412	0.8569	0.7918	0.6595	0.8864

Table 1.9: Estimates of $\rho = 0.9$

lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
	T=400								
	Mix PMLE			**OLS**			**Cauchy PMLE**		
0.0000	1.1104	1.1018	1.1124	0.8948	0.8564	0.9242	1.1104	1.1018	1.1124
0.0100	1.1104	1.1010	1.1123	0.8947	0.8552	0.9247	1.1105	1.1010	1.1126
0.1000	1.1102	1.1003	1.1124	0.8950	0.8582	0.9254	1.1103	1.1002	1.1123
0.2000	1.1100	1.0979	1.1125	0.8952	0.8580	0.9261	1.1100	1.0991	1.1125
0.3000	1.1097	1.0970	1.1123	0.8951	0.8572	0.9237	1.1098	1.0954	1.1127
0.4000	1.1093	1.0950	1.1122	0.8950	0.8588	0.9278	1.1095	1.0906	1.1125
0.5000	1.1086	1.0862	1.1123	0.8950	0.8588	0.9253	1.1090	1.0883	1.1125
0.6000	1.1075	1.0784	1.1121	0.8952	0.8613	0.9306	1.1080	1.0756	1.1126
0.7000	1.1042	1.0574	1.1119	0.8950	0.8527	0.9233	1.1049	1.0581	1.1122
0.8000	1.0963	1.0252	1.1114	0.8952	0.8564	0.9245	1.0987	1.0192	1.1120
0.8500	1.0892	0.9894	1.1115	0.8944	0.8540	0.9327	1.0881	0.9902	1.1116
0.9000	1.0699	0.9445	1.1110	0.8944	0.8506	0.9255	1.0580	0.9351	1.1112
0.9500	0.9961	0.8915	1.1093	0.8927	0.8499	0.9242	0.9913	0.8908	1.1098
1.0000	0.8977	0.8580	0.9283	0.8905	0.8441	0.9241	0.8984	0.8478	0.9320
lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
	T=100								
	Mix PMLE			**OLS**			**Cauchy PMLE**		
0.0000	1.1087	1.0732	1.1164	0.8776	0.7758	0.9997	1.1087	1.0732	1.1164
0.0100	1.1086	1.0728	1.1165	0.8776	0.7724	0.9945	1.1086	1.0753	1.1163
0.1000	1.1083	1.0664	1.1169	0.8783	0.7713	1.0117	1.1080	1.0720	1.1160
0.2000	1.1076	1.0656	1.1165	0.8769	0.7637	1.0161	1.1075	1.0683	1.1158
0.3000	1.1064	1.0615	1.1159	0.8772	0.7767	0.9962	1.1069	1.0544	1.1180
0.4000	1.1047	1.0495	1.1161	0.8774	0.7630	1.0175	1.1052	1.0421	1.1169
0.5000	1.1018	1.0274	1.1149	0.8774	0.7787	1.0139	1.1021	1.0239	1.1178
0.6000	1.0984	1.0127	1.1148	0.8770	0.7678	1.0001	1.0993	1.0001	1.1164
0.7000	1.0929	0.9621	1.1150	0.8769	0.7642	0.9818	1.0884	0.9624	1.1159
0.8000	1.0708	0.9147	1.1148	0.8758	0.7592	0.9602	1.0680	0.9060	1.1147
0.8500	1.0477	0.8818	1.1127	0.8771	0.7697	0.9756	1.0514	0.8785	1.1135
0.9000	0.9923	0.8453	1.1117	0.8765	0.7722	0.9655	1.0054	0.8493	1.1118
0.9500	0.9334	0.8219	1.1068	0.8744	0.7540	0.9481	0.9429	0.8148	1.1073
1.0000	0.8898	0.7848	0.9451	0.8708	0.7678	0.9356	0.8915	0.7887	0.9551