Tests for No-Arbitrage in Cryptocurrency Markets

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Abstract

This article investigates the efficiency of the cryptocurrency markets by testing the no-arbitrage hypothesis in a general framework for cryptocurrencies traded on multiple exchange platforms. We test the moment conditions implied by the no-arbitrage hypothesis by applying a simulated method of moments and a general specification test. For each of the five cryptocurrencies considered, the test is applied globally using the full sample and locally using a rolling window approach. Globally, the results show that the null hypothesis of no-arbitrage is rejected for Bitcoin and Ethereum, but is not rejected for Binance Coin and for the two stablecoins, USD Coin and Tether. Locally, the hypothesis is never rejected for USD Coin and is rejected 5.5% of the time for Tether. In comparison, the test rejects the hypothesis frequently for the three studied native cryptocurrencies (39.5% of the time for Bitcoin, 26% for Ethereum, and 21.5% for Binance Coin).

Keywords: cryptocurrency, market efficiency, simulated method of moments, Wishart autoregressive process, double-autoregressive process.

1 Introduction

There is an ongoing debate concerning the efficiency of the cryptocurrency markets. Several articles have examined the market efficiency of cryptocurrencies such as Bitcoin, Ethereum, Ripple, Litecoin and EOS and provide evidence of time varying market efficiency of cryptocurrency [see, e.g., Tran, Leirvik (2020), Dhawan, Putnins (2020), Liu, Tsyvinski, Wu (2019), Chu, Zhang, Chan (2019), Li, Shin, Wang (2018)]. While the literature seems to agree that prior to 2017 the cryptocurrency markets were mostly efficient [see, e.g. Urquhart (2016), Vidal-Tomas, Ibanez (2018), Zargar, Kumar (2019)], there is no consensus regarding the period from 2017 to the present. For example, Tran, Leirvik (2020) found evidence of market efficiency over this period, while other authors conclude that the cryptocurrency markets were largely inefficient after 2017.

This paper studies the efficiency of the cryptocurrency markets by testing the no-arbitrage hypothesis in a framework similar to that in Gourieroux, Monfort, Sufana (2010), which was developed as a unified approach for pricing bonds, stocks, currencies, and their derivatives and used the Wishart autoregressive (WAR) process (Gourieroux, Jasiak, Sufana , 2009) as a model for fundamental risk affecting all assets of interest. Wishart processes are tractable modeling tools since they are special cases of affine processes (see (Duffie, Kan , 1996) and (Duffie, Filipovic, Schachermayer , 2003)), and have been used in the financial literature to model multivariate risk, in particular stochastic covariance or correlation matrices, in various applications, such as applications to derivative pricing (Gourieroux, Sufana , 2010) and portfolio choice (Buraschi, Porchia, Trojani , 2010).

We extend the applicability of this framework by using it as a general approach for modeling cryptocurrencies traded on multiple exchange platforms. To the best of our knowledge, this paper is the first to analyze the pricing of major cryptocurrencies and their efficiency in such a general framework. We assume that the cryptocurrency assets are driven by a common fundamental risk factor and idiosyncratic risk factors specific to each asset. The dynamics of the fundamental risk factor is modeled using a univariate WAR process, while the idiosyncratic risk factors are assumed to follow a conditional multivariate Gaussian distribution. In the univariate case, the WAR process reduces to an autoregressive gamma process (Gourieroux, Jasiak , 2006), and is a discretized version of the CIR process (Cox, Ingersoll, Ross , 1981) used in the literature to study short-term interest rates or univariate stochastic volatility.

The simulated method of moments facilitates the estimation of the parameters in the model specification. In particular, this estimation method helps avoid maximizing a complex likelihood function and instead relies on maximizing an objective function that captures the distance between sample moments and the simulated counterparts. The no-arbitrage hypothesis implies a set of pricing restrictions in the form of moment conditions. We test these moment conditions by applying a general specification test.

To account for possible persistence in the autoregressive (AR) component of fundamental risk factor specification, we expand our framework by replacing it with a double-autoregressive (DAR) process. For instance, in Djogbenou, Inan, Jasiak (2023), this process provides greater flexibility to handle some non-stable patterns that might occur over time when modeling returns of crypto-assets. In addition to the autoregressive coefficient that allows for capturing the conditional mean dependence, the DAR processes usually incorporate a parameter representing the past dependence in the conditional variance of the considered variable. More details on these processes can be found in Borkovec, Kluppenberg (2001), Ling (2004), Ling (2007), and Ling, Li (2008). The considered extension combines the important features of the univariate WAR and the DAR processes.

We consider the largest five cryptocurrencies by market capitalization on November 16, 2022, represented by Bitcoin (BTC), Ethereum (ETH), Tether (USDT), Binance Coin (BNB), and USD Coin (USDC). For each cryptocurrency, the top six trading pairs by daily volume are selected, and the test of the no-arbitrage hypothesis is applied first globally using the full sample, and then locally using a rolling window approach. Globally, the results show that the null hypothesis of no-arbitrage is rejected for Bitcoin and Ethereum, but is not rejected for Binance Coin and for the two stablecoins, USD Coin and Tether. Locally, we find that there are no arbitrage opportunities for USD Coin and a very limited number of such opportunities for Tether. In contrast, the results show that there are frequent arbitrage opportunities for the three native cryptocurrencies. Specifically, the percentage of days with no arbitrage opportunities is 100% for USD Coin, 94.5% for Tether, 60.5% for Bitcoin, 74% for Ethereum, and 78.5% for Binance Coin.

This paper is organized as follows. Section 2 describes the setup, the assumptions, and the no-arbitrage hypothesis. Section 3 describes the model estimation, the test for no-arbitrage, and the order condition for identification. Section 4 describes the cryptocurrency markets and the data, and presents the empirical results. Tables for the standard approach based on the univariate WAR are relegated to Appendix A.

2 The Model

We use a model for a set of n cryptocurrencies that is similar to the framework described in Gourieroux, Monfort, Sufana (2010), and choose the US dollar as the numeraire currency.

2.1 Setup and Assumptions

We assume that the information available to investors includes a latent fundamental risk factor, whose value at time t is denoted Σ_t, and some idiosyncratic risk factors, given by the $(n, 1)$ vector $\epsilon_t = (\epsilon_{1,t}, ..., \epsilon_{n,t})'$, where $\epsilon_{j,t}$ is the idiosyncratic risk of cryptocurrency j at time t, for $j = 1, ..., n$.

The latent factor Σ_t is assumed to follow a univariate Wishart autoregressive (WAR) process (Gourieroux, Jasiak, Sufana , 2009) with one degree of freedom, which is also a special case of autoregressive gamma process (Gourieroux, Jasiak , 2006). The process can be expressed as

$$\Sigma_t = x_t^2, \tag{2.1}$$

where x_t is a Gaussian autoregressive process

$$x_t = \mu x_{t-1} + \eta_t, \tag{2.2}$$

with latent autoregressive coefficient μ and latent innovation variance Ω, where $\eta_t \sim N(0, \Omega)$. We extend the specification of η_t in Section 4.2.1 to make x_t a double-autoregressive process.

The vector ϵ_{t+1} of idiosyncratic risks is assumed to be multivariate Gaussian conditional on $\underline{\Sigma_{t+1}}$, $\underline{\epsilon_t}$ with mean zero and variance matrix $A'\Sigma_{t+1}A$, where $\underline{\Sigma_{t+1}} = (\Sigma_{t+1}, \Sigma_t, ...)$, $\underline{\epsilon_t} = (\epsilon_t, \epsilon_{t-1}, ...)$, and A is a $(1, n)$ vector.

For $j = 1, ..., n$, let $S_{j,t}$ denote the price of cryptocurrency j at time t, and let $y_{j,t+1} = ln\ S_{j,t+1} - ln\ S_{j,t}$ denote its log return for the period $(t, t+1)$. The dynamics of $y_{j,t+1}$ is specified as an affine function of the fundamental and the idiosyncratic risk factors:

$$y_{j,t+1} = f_j + F_j^0 \Sigma_t + \epsilon_{j,t+1}, \tag{2.3}$$

where f_j and F_j^0 are scalars, with $F_j^0 \geq 0$ since we expect the risk premium $F_j^0 \Sigma_t$ to be positive.

2.2 No-arbitrage Hypothesis

The hypothesis of absence of arbitrage opportunities implies the existence of a stochastic discount factor (sdf), which can be used to price all tradable assets of interest, including cryptocurrencies. We assume that the sdf for the period $(t, t+1)$ is an exponential affine function of the fundamental and idiosyncratic risk factors:

$$M_{t,t+1} = exp\left(c + C^0 \Sigma_t + \gamma' \epsilon_{t+1}\right), \tag{2.4}$$

where c and C^0 are scalars and γ is an $(n, 1)$ vector.

Similar specifications were also considered by Gourieroux, Monfort (2007). The coefficient C^0 and the vector γ capture the sensitivities of the sdf to the factors. As in Gourieroux, Monfort, Sufana (2010), we expect $C^0 \leq 0$, implying that the state prices decrease with the fundamental risk.

Under the no-arbitrage hypothesis, we have the following pricing restrictions:

$$E_t\left[M_{t,t+1} S_{j,t+1}\right] = S_{j,t}, \tag{2.5}$$

for $j = 1, ..., n$, which are equivalent to

$$E_t \left[M_{t,t+1} exp\ y_{j,t+1} \right] = 1, \tag{2.6}$$

for $j = 1, ..., n$, where E_t denotes the expectation conditional on the information set $\underline{\Sigma_t}$, $\underline{\epsilon_t}$. The following section discusses the estimation of the different parameters and how the no-arbitrage hypothesis can be tested.

3 Simulated Method of Moments

Originally proposed by McFadden (1989), the simulated method of moments (SMM) is a form of generalized method of moments where the parameter estimators are chosen such that simulated model moments match data moments. Under fairly general regularity conditions, Duffie, Singleton (1993) show the consistency and asymptotic normality of the SMM estimators. Further discussions of the SMM can be found in Gourieroux, Monfort, Renault (1993), Gallant, Tauchen (1996), Gourieroux, Monfort (2002), Davidson, MacKinnon (2004), and Ruge-Murcia (2012).

3.1 Model Estimation and Test for No-arbitrage

Let a vector $\theta : P \times 1$ collect the vectorized version of all unknown parameters $M, \Omega, f_1, \ldots, f_n, F_1^0, \ldots, F_n^0, A, c, C^0$, and γ. For any admissible parameter vector θ in a compact set Θ, we can obtain the model-based simulated version of the vector of cryptocurrencies' returns $y_{s+1}^\theta = \left(y_{1,s+1}^\theta, \ldots, y_{n,s+1}^\theta\right)'$, using

$$y_{j,s+1}^\theta = f_j + F_j^0 \Sigma_s^\theta + \epsilon_{j,s+1}^\theta, s = 1, \ldots, S, \tag{3.1}$$

where $S = \tau T$ is the number of times the returns are generated and $\tau \geq 1$. The factor Σ_s^θ can be generated using Equation 2.1:

$$\Sigma_s^\theta = x_s^2, \tag{3.2}$$

and using Equation 2.2:

$$x_s^\theta = \mu x_{s-1}^\theta + \eta_s^\theta, \tag{3.3}$$

$s = 1, \ldots, S$. The innovation $\epsilon_{j,s+1}^\theta$ is a typical element of ϵ_{s+1}^θ drawn from a multivariate Gaussian with mean $0_{n\times1}$ and variance $A'\Sigma_{s+1}^\theta A$ conditional on $\underline{\Sigma_{s+1}^\theta} = (\Sigma_{s+1}^\theta, \Sigma_s^\theta, \ldots)$ and $\underline{\epsilon_s^\theta} = (\epsilon_s^\theta, \epsilon_{s-1}^\theta, \ldots)$, while η_s^θ is drawn from a $N(0, \Omega)$ distribution.

To construct our objective function for estimating the unknown parameters, we note from the no-arbitrage condition in Equation (2.6) that

$$E_t\left[y_{j,t-q}M_{t,t+1}\exp\ y_{j,t+1}\right]=y_{j,t-q}, j=1,\dots,n, q=1,\dots,Q.$$

By the law of iterated expectation, we have:

$$E(y_{j,t+1-q})=E\left[y_{j,t+1-q}M_{t,t+1}\exp\ y_{j,t+1}\right], j=1,\dots,n, q=1,\dots,Q. \tag{3.4}$$

We can rewrite Equation 3.4 as

$$E\underbrace{\begin{pmatrix} y_{1,t} \\ y_{2,t} \\ \vdots \\ y_{n,t} \\ \vdots \\ y_{1,t+1-q} \\ y_{2,t+1-q} \\ \vdots \\ y_{j,t+1-q} \\ \vdots \\ y_{1,t+1-Q} \\ y_{2,t+1-Q} \\ \vdots \\ y_{j,t+1-Q} \end{pmatrix}}_{z_t} = E\underbrace{\begin{pmatrix} y_{1,t}M_{t,t+1}y_{1,t+1} \\ y_{2,t}M_{t,t+1}y_{2,t+1} \\ \vdots \\ y_{n,t}M_{t,t+1}y_{n,t+1} \\ \vdots \\ y_{1,t+1-q}M_{t,t+1}y_{1,t+1} \\ y_{2,t+1-q}M_{t,t+1}y_{2,t+1} \\ \vdots \\ y_{n,t+1-q}M_{t,t+1}y_{n,t+1} \\ \vdots \\ y_{1,t+1-Q}M_{t,t+1}y_{1,t+1} \\ y_{2,t+1-Q}M_{t,t+1}y_{2,t+1} \\ \vdots \\ y_{n,t+1-Q}M_{t,t+1}y_{n,t+1} \end{pmatrix}}_{Z_{t+1}} \tag{3.5}$$

So, Equation 3.5 is equivalent to $E(z_t)=E(Z_{t+1})$, where z_t and Z_{t+1} are nQ-vectors depending on information up to time t and $t+1$, respectively. Denote by Z^{θ}_{s+1}, the simulated version of Z_{t+1}, which replaces $M_{t,t+1}, y_{j,t+1}, y_{j,t+1-q}$, with $M^{\theta}_{s,s+1}, y^{\theta}_{j,s+1}, y^{\theta}_{j,s+1-q}$, where

$$M^{\theta}_{s,s+1}=\exp(c+C^0\Sigma^{\theta}_s+\gamma'\epsilon^{\theta}_{s+1}). \tag{3.6}$$

The SMM matches the sample moment

$$g^*_T=\frac{1}{T}\sum_{t=1}^{T}z_t,$$

the sample analog of $E(z_t)$ based on observed data, to

$$g_S(\theta)=\frac{1}{S}\sum_{s=1}^{S}Z^{\theta}_{s+1},$$

the simulation-based sample analog of $E(Z_{t+1})$. The SMM estimator is given by

$$\hat{\theta} = \arg\min_{\theta\in\Theta} G(\theta)'\hat{V}^{-1}G(\theta), \tag{3.7}$$

where

$$G(\theta) = g_T^* - g_S(\theta) \tag{3.8}$$

with $\hat{V}$, any consistent estimator of the asymptotic variance of $\sqrt{T}G(\theta)$. An heteroskedasticity and autocorrelation consistent (HAC) estimator for V is given by

$$\hat{V} = \frac{1}{T}\sum_{t=1}^{T} u_t u_t' + \frac{1}{T}\sum_{h=1}^{B_T}\sum_{t=h+1}^{T} k\left(\frac{h}{B_T}\right)(u_t u_{t-h}' + u_{t-h}u_t'), \tag{3.9}$$

where $k(\cdot)$ is a kernel function, B_T is the bandwidth, and $u_t = z_t - \frac{1}{T}\sum_{t=1}^{T} z_t$, similarly to Newey, West (1987). Under mild conditions, it follows from Lee, Ingram (1991) that the simulated method of moment estimator $\hat{\theta}$ is asymptotically normal with the asymptotic distribution given by

$$\sqrt{T}(\hat{\theta} - \theta) \xrightarrow{d} N\left(0, \left(1 + \frac{1}{\tau}\right) BV^{-1}B'\right), \tag{3.10}$$

where $B = E\left(\frac{\partial(Z_{t+1}^{\theta})'}{\partial\theta}\right)$ and $\left(1 + \frac{1}{\tau}\right) BV^{-1}B'$ corresponds to the asymptotic variance formula based on the optimal weighting matrix $\hat{V}^{-1}$ minimizing the objective function in Equation 3.7. The variance in the asymptotic distribution in Equation 3.10 shows that the importance of the randomness due to the simulation reduces as τ increases.[1]

To test for no-arbitrage, we test the validity of the moment conditions in Equation 3.5. Therefore, we can test if the identification restrictions generated by the no-arbitrage assumption are valid. In consequence, we can use the Hansen's general specification test defined by

$$W_T = T\left(1 + \frac{1}{\tau}\right) G(\hat{\theta})'\hat{V}^{-1}G(\hat{\theta}), \tag{3.11}$$

where

$$W_T \xrightarrow{d} \chi^2(nQ - P), \tag{3.12}$$

with P, the number of parameters, nQ, the number of conditions used, and $nQ > P$. See, e.g., Hansen (1982) and Ruge-Murcia (2012) for more details on the general description of the test and the simulated method of moments, respectively.

[1]In our application, we set τ to 20, and find this choice to be reasonable after sensitivity analyses, while taking into account the need to have a large number of simulated data.

3.2 Order Condition

In this subsection, we discuss the order condition for the SMM estimators. This condition is derived based on the number nQ of moment conditions in Equation 3.4 that are used for estimation and the number P of model parameters, where Q denotes the number of lags.

We first note that Σ_s^θ depends on the latent autoregressive coefficient μ and the latent innovation variance Ω. Therefore, we can simulate Σ_s^θ based on 2 parameters. Second, if we know Σ_s^θ and the parameters in the $(1, n)$ vector A, then we can simulate ϵ_s^θ. Third, using $2n$ parameters for f_j and F_j^0, $j = 1, ..., n$, we can also simulate the cryptoassets returns $y_{j,s}^\theta$. In addition, we need $n + 2$ parameters for c, C^0, and γ in the expression of the sdf $M_{s,s+1}^\theta$. Therefore, we have to estimate a total of $P = 4n + 4$ parameters.

Since we have P parameters and nQ moment conditions, the number n of cryptoassets needs to be such that $nQ \geq P$, in order to be able to identify the model. Moreover, this inequality needs to be strict, for the degrees of freedom of the asymptotic distribution in 3.12 to be strictly positive. This result is summarized in the proposition below.

Proposition 1 *For the estimation of the unknown parameters, the number n of cryptoassets has to satisfy the order condition:*

$$nQ > 4n + 4. \tag{3.13}$$

The proposition implies that we need the number of lags $Q \geq 5$, because $Q > \frac{4}{n} + 4$. Also, for $Q = 5$, we need the number of cryptoassets $n \geq 5$.

4 Empirical Application

This section describes the cryptocurrency markets and the data, and presents the empirical results.

4.1 The cryptocurrency markets and the data

Crypto exchange platforms play an important role in facilitating day-to-day transactions for cryptocurrency investors. Through an exchange platform, investors can transfer funds to get in and out of the cryptomarket, trade one cryptocurrency for another, and store their funds securely in crypto wallets. As cryptocurrency exchange platforms exist only online, investors need to create an account on exchange platform's website to start using their services.

For an investor who wants to actively trade in the cryptomarket, there are mainly two types of exchange platforms they can choose from including centralized exchanges (CEXs) and decentralized exchanges (DEXs). The main difference between these two exchange types is the way the trade orders placed by market participants are carried out. When a trade order is executed under the supervision of a central authority, an exchange platform is considered as centralized. In decentralized exchanges, trade orders are executed with the help of automated algorithms on blockchain technology, which allows users to engage in peer-to-peer transactions without the need for a third party.

Despite the increasing number of decentralized exchanges, the cryptomarket is still vastly dominated by centralized ones as far as the daily trade volume is concerned. According to the recent research by Citi,[2] decentralized exchanges account for only 18.2% of the spot-trading volume with Uniswap itself, currently the largest decentralized exchange in the market, making up 70% of the total DEX volume.

For the analysis, we restrict our sample to the data from the five most reliable cryptocurrency exchanges in terms of the exchange score[3] according to coinmarketcap.com as of November 16, 2022. The set of exchange platforms includes Binance, Coinbase Exchange, Kraken, Kucoin, and Binance.US. and each platform in this set has a score of at least 7 (out of 10). These exchange platforms cover a significant part of the overall trading volume and are used to proxy the cryptocurrency markets.[4]

The reason why we select exchange platforms based on their reliability instead of their size is because daily volumes reported by exchange platforms may not always be accurate. In fact, some of the big exchange markets are found to have engaged in wash trading meaning that they fake their trading volume to improve their ranking and popularity in the industry (Cong, Li, Tang, Yang (2022); Le Pennec, Fiedler, Ante (2021)) and manipulate cryptocurrency prices as documented by Chen, Lin, Wu (2022).

Table 1: The top 5 cryptocurrency exchanges by reliability score as of November 16, 2022

Exchange Platform	# Cryptocurrencies	# Trading Pairs	Fiat Supported
Binance	386	1691	USD, EUR, GBP and +43 more
Coinbase Exchange	229	595	USD, EUR, GBP
Kraken	217	713	USD, EUR, GBP and +4 more
KuCoin	766	1389	USD, EUR, GBP and +45 more
Binance.US	145	309	USD

Source: https://coinmarketcap.com/rankings/exchanges/

[2] https://www.coindesk.com/business/2022/10/03/citi-says-decentralized-crypto-exchanges-are-winning-market-share-from-centralized-peers/

[3] Coinmarketcap.com states that the exchange score is calculated as the weighted average of web traffic factor, average liquidity, volume, and the confidence that the volume reported by an exchange is legitimate.

[4] The number of exchanges is large enough to satisfy the order condition in Proposition 1.

All cryptocurrency exchanges in our sample are centralized so their business models are very similar. However, these exchange platforms can still be distinguished from one another when it comes to the number of cryptocurrencies that can be bought and sold, the list of trading pairs available to investors, and the set of fiat currencies supported by the platform. Table 1 summarizes the current state of the exchange platforms in our sample with respect to these characteristics.

4.1.1 Data Description

We consider the five largest cryptocurrencies by market capitalization including Bitcoin (BTC), Ethereum (ETH), Tether (USDT), Binance Coin (BNB), and USD Coin (USDC). Then, we rank all trading pairs of these cryptocurrencies with respect to their daily volume as of November 16, 2022 and then select the top six pairs for each cryptocurrency. This adds up to a total of 30 different prices series that will be used in the analysis. The complete list of the trading pairs can be found in Table 2 along with the names of the exchange platforms each pairs were traded on, the respective daily volumes in US dollars, and their percentage contributions to the total daily volume.

The historical data on daily closing price of each trading pairs is obtained directly from the exchange platforms via their Application Programming Interfaces (APIs). It is worth noting that exchange platforms store and publish the historical data on trading pairs in such a way that the price of the base cryptocurrency is usually expressed in terms of the units of the quote cryptocurrency. For consistency, we convert the price of all trading pairs into US dollars by using the corresponding exchange rate between the quote cryptocurrency and the US dollar retrieved from Yahoo Finance.

Table 3 provides the summary statistics of daily log returns of cryptocurrencies with respect to trading pairs.

Table 2: List of cryptocurrency trading pairs with the highest daily volumes as of November 16, 2022

Cryptocurrency	Trading Pair	Exchange Platform	Daily Volume	Daily Volume (%)
Binance Coin (BNB)	BNB/USDT	Binance	$117,548,065	12.20%
	BNB/BUSD	Binance	$95,634,654	9.92%
	BNB/BTC	Binance	$15,580,100	1.62%
	BNB/ETH	Binance	$5,083,991	0.53%
	BNB/USDT	KuCoin	$3,500,603	0.36%
	BNB/EUR	Binance	$3,182,434	0.33%
Bitcoin (BTC)	BTC/USDT	Binance	$4,738,786,091	13.16%
	BTC/BUSD	Binance	$2,722,430,698	7.56%
	BTC/USD	Coinbase Exchange	$807,460,570	2.24%
	BTC/USDT	KuCoin	$203,840,895	0.57%
	BTC/USD	Binance.US	$166,547,193	0.46%
	ETH/BTC	Binance	$136,816,282	0.38%
Ethereum (ETH)	ETH/USDT	Binance	$1,044,487,266	7.17%
	ETH/USD	Coinbase Exchange	$755,769,692	5.19%
	ETH/BUSD	Binance	$645,680,778	4.43%
	ETH/USDT	KuCoin	$141,319,928	0.97%
	ETH/BTC	Binance	$137,002,090	0.94%
	ETH/USD	Kraken	$91,258,418	0.63%
Tether (USDT)	BTC/USDT	Binance	$4,738,786,091	9.38%
	BUSD/USDT	Binance	$1,151,360,536	2.28%
	ETH/USDT	Binance	$1,044,238,487	2.07%
	USDT/USD	Kraken	$299,184,079	0.59%
	BTC/USDT	KuCoin	$203,840,895	0.40%
	XRP/USDT	Binance	$198,081,627	0.39%
USD Coin (USDC)	USDC/USD	Kraken	$58,226,926	1.30%
	USDC/USDT	Kraken	$33,092,103	0.74%
	USDC/EUR	Kraken	$24,085,577	0.54%
	USDC/USDT	KuCoin	$9,354,817	0.21%
	BTC/USDC	KuCoin	$9,246,101	0.21%
	ETH/USDC	KuCoin	$8,027,523	0.18%

Table 3: Summary statistics of daily log returns until November 16, 2022

Trading Pair	**Beginning Date**	**Number of Days**	**Min**	**Max**	**Mean**	**Std**
BNB/BTC	2017-07-14	1958	-0.549	0.657	0.0040974	0.0688291
BNB/BUSD	2019-09-20	1160	-0.527	0.534	0.0021692	0.0558232
BNB/ETH	2017-11-09	1840	-0.545	0.536	0.0026349	0.0595479
BNB/EUR	2020-01-03	1055	-0.588	0.531	0.0028154	0.0571841
BNB/USDT	2017-11-06	1843	-0.529	0.533	0.0026505	0.0603182
BNB/USDT	2019-06-19	1253	-0.531	0.536	0.0016067	0.0551446
BTC/USDT	2017-11-09	1840	-0.449	0.224	0.0004430	0.0412449
BTC/BUSD	2019-09-20	1160	-0.449	0.178	0.0003966	0.0390128
BTC/USD	2017-02-23	2099	-0.491	0.241	0.0012454	0.0421090
BTC/USDT	2018-07-08	1599	-0.448	0.178	0.0005486	0.0386145
BTC/USD	2019-09-24	1156	-0.484	0.178	0.0005489	0.0395931
ETH/BTC	2017-11-09	1840	-0.460	0.227	0.0004411	0.0405444
ETH/USDT	2017-11-09	1840	-0.537	0.255	0.0006837	0.0527709
ETH/USD	2017-02-23	2099	-0.568	0.282	0.0021212	0.0562751
ETH/BUSD	2019-10-21	1129	-0.540	0.235	0.0016527	0.0518516
ETH/USDT	2018-07-08	1599	-0.537	0.234	0.0005253	0.0511201
ETH/BTC	2017-07-14	1958	-0.555	0.247	0.0008752	0.0533127
ETH/USD	2015-10-11	2600	-0.583	0.372	0.0028438	0.0595864
BTC/USDT	2017-08-17	1924	-0.069	0.063	-0.0000061	0.0061191
BUSD/USDT	2019-09-20	1160	-0.050	0.053	-0.0000017	0.0033486
ETH/USDT	2017-11-09	1840	-0.063	0.057	-0.0000065	0.0067026
USDT/USD	2017-03-29	2065	-0.058	0.045	-0.0000001	0.0043200
BTC/USDT	2018-07-08	1599	-0.038	0.037	-0.0000093	0.0042588
XRP/USDT	2018-05-04	1664	-0.029	0.037	-0.0000012	0.0051878
USDC/USD	2020-01-08	1050	-0.010	0.010	-0.0000096	0.0007398
USDC/USDT	2020-01-08	1050	-0.057	0.048	-0.0000216	0.0034762
USDC/EUR	2020-01-08	1050	-0.031	0.042	-0.0000242	0.0061821
USDC/USDT	2020-11-23	730	-0.012	0.010	0.0000028	0.0010906
BTC/USDC	2019-01-07	1416	-0.173	0.149	-0.0000105	0.0080633
ETH/USDC	2019-01-07	1416	-0.167	0.123	-0.0000451	0.0096427

4.2 Empirical Results

This section presents the estimation results using the observed daily returns of the cryptocurrencies. Given that the digital currencies can have different returns on different platforms, we treat the cryptocurrencies on different platforms as different assets. Using the data, we answer the following empirical questions. To which extent are the cryptocurrencies priced and affected by the fundamental risk factor? Are individual cryptocurrency markets free of arbitrage opportunity?

To answer these questions, for each cryptocurrency (Bitcoin (BTC), Ethereum (ETH), Tether (USDT), Binance Coin (BNB), and USD Coin (USDC)), we apply the SMM using the corresponsing six trading pair samples ($n = 6$). We choose the number of lags $Q = 5$, so that the values of n and Q satisfy the order condition in Section 3.2.

To implement the SMM, we proceed as follows.

1. Set initial values for the parameters: $\mu, \Omega, f_1, \dots, f_n, F_1^0, \dots, F_n^0, A, c, C^0$, and γ. We consider 1000 sets of initial values and select the set that leads to the lowest value of the objective function in Equation 3.7.

2. Simulate the fundamental risk factor $\Sigma_{s+1}^{\theta}, s = 1, \dots, S$, using Equation 3.2 and Equation 3.3.

3. For $s = 1, \dots, S$, generate the $(n, 1)$ vector ϵ_{s+1}^{θ} of idiosyncratic risks of cryptoassets from a multivariate Gaussian distribution with conditional mean 0_n and conditional variance $A'\Sigma_{s+1}^{\theta}A$.

4. For $s = 1, \dots, S$, compute $y_{j,s+1}^{\theta}, j = 1, \dots, n$ and $M_{s,s+1}^{\theta}$ based on Equations 3.1 and 3.6, respectively, and using Σ_{s+1}^{θ} and ϵ_{s+1}^{θ}.

5. Find $\hat{\theta}$ from the optimization problem in Equation 3.7, based on the generated $y_{j,s+1}^{\theta}, j = 1, \dots, n$ and $M_{s,s+1}^{\theta}$. Obtain the asymptotic normal interval for the components of $\hat{\theta}$.

6. Perform the no-arbitrage test using Equations 3.11 and 3.12.

The variance $\hat{V}$ employed in Steps 5 and 6 is computed using a Bartlett kernel with the bandwidth, which we set to $4(T/100)^{2/9}$, as suggested by Newey, West (1994). The kernel $k(\cdot)$ is a decreasing function, which accounts for the the decaying dependence between the observations at t and $t + h$ as h increases.

4.2.1 Global Analysis of Individual Cryptocurrency Markets

We first conduct a global analysis using the sample data over the last two years, which consists of 730 days, in order to minimize the impact of changes in the market's structure over multiple years. The model parameters are estimated for all cryptocurrencies. The results are presented in Table A.1 in Appendix A. The table shows that the absolute value of the latent autoregressive coefficient μ is estimated to be 1 or very close to 1 for Binance Coin, Bitcoin, Ethereum, and USD Coin, implying that the corresponding processes for fundamental risk are nonstationary and, therefore, casting some doubt on the validity of the estimation results.

In response to this finding, we extend our analysis by considering a modified version of the standard model where we assume that x_t in Equation 2.1 follows the process:

$$x_t = \mu x_{t-1} + \eta_t,$$

with $\eta_t \sim N(0, w + \alpha x_{t-1}^2)$, where w and α are two new parameters. This specification is similar to the one used in Djogbenou, Inan, Jasiak (2023) and we refer to it as the DAR version of the model. In this case, the total number of parameters is $4n + 5$ and the order condition in Equation (3.13) becomes

$$nQ > 4n + 5.$$

This condition holds when $Q = 5$ and $n = 6$.

The SMM is implemented in a very similar manner with minor changes. In particular, η_s^θ is simulated from a normal distribution, with the difference that Ω is replaced by $w+\alpha(x_{s-1}^\theta)^2$. The point estimates for the coefficients of the DAR model are reported in Table 4. As seen in the table, for each cryptocurrency, the estimate of the latent autoregressive coefficient M implies a more realistic fundamental risk process.

For both Table A.1 in Appendix A and Table 4, we also note that most estimates of the coefficient F^0 in the cryptoasset return equation are positive. As a result, the corresponding returns (and risk premia) depend positively on the fundamental risk Σ_t. Also, all estimates of the coefficient C^0 in the stochastic discount factor are negative, showing that the state prices decrease with the fundamental risk.

For the global analysis of each cryptocurrency, the result of the no-arbitrage test along with the estimated test statistics $\hat{W}_T$ and the critical value for the 95% confidence level based on Equation 3.11 and Equation 3.12, respectively, are summarized in Table A.2 (for the standard AR model) and Table 5 (for the DAR model). We consider the results for the DAR model in Table 5 as our official test results and present the results in Table A.2 for an interesting comparison. Even if the test results in Table A.2 are technically less reliable (for the reason discussed above), these results are the same as those in Table 5 for four of the five cryptocurrencies.

The tables show that, for Bitcoin and Ethereum, the null hypothesis that there are no-arbitrage opportunities across trading pairs is rejected in both the standard AR model and the DAR model. These findings suggest that, for these cryptocurrencies, investors could profit by buying and selling different trading pairs of the same cryptocurrency. At the same time, for USD Coin and Tether, the tables indicate that the null hypothesis is not rejected in both models. This is not surprising since these cryptocurrencies are designed to maintain a stable price against the US dollar. Hence, they are often referred to as stablecoins. For Binance Coin, the null hypothesis is rejected in the standard AR model but is not rejected in the DAR model.

Table 4: DAR: Global Parameter Estimates

Model Parameters	Binance Coin (BNB)		Bitcoin (BTC)		Ethereum (ETH)		USD Coin (USDC)		Tether (USDT)	
	Point Estimate	***Standard Deviation***	***Point Estimate***	***Standard Deviation***	***Point Estimate***	***Standard Deviation***	***Point Estimate***	***Standard Deviation***	***Point Estimate***	***Standard Deviation***
μ	0.142***	0.002	0.133***	0.005	0.021***	1.172E-08	0.161***	5.250E-05	0.204***	3.849E-05
w	8.345E-06	305.244	2.678***	0.001	0.001***	9.168E-05	0.660***	9.041E-05	0.010	1.031E-02
α	0.053***	0.004	0.128***	0.004	0.129***	3.106E-08	0.051***	4.280E-06	0.031***	2.124E-04
f_1	-0.045	7.496	-0.112***	0.020	10.061***	2.726E-08	-4.594***	1.994E-06	0.588***	1.716E-02
f_2	-0.045	13.461	-0.321***	0.003	4.912***	1.007E-10	-4.547***	4.423E-07	0.117***	5.589E-04
f_3	-0.045	6.238	0.107***	0.012	-2.491***	1.625E-13	-15.032***	6.587E-12	0.793***	3.658E-03
f_4	-0.047	1.362	-0.032	0.020	6.370***	3.979E-09	-1.243***	1.918E-07	0.317***	3.067E-04
f_5	-0.045	28.956	-0.394***	0.009	-4.349***	5.139E-14	-1.165***	6.688E-07	0.551***	1.568E-02
f_6	-0.045	28.750	-0.844***	0.002	8.383***	9.848E-09	0.337***	3.228E-05	-0.104***	5.110E-04
F_1^0	0.597***	7.613E-05	0.088*	0.046	3.169***	1.335E-11	7.554E-08	3.313E-07	6.481E-05	2.010E-04
F_2^0	2.26E-04*	1.366E-04	0.113***	0.008	1.128***	6.966E-14	0.481***	8.575E-08	0.523***	6.662E-06
F_3^0	0.422***	6.354E-05	2.480E-08	0.031	0.251***	3.312E-17	0.107***	1.006E-12	0.325***	4.340E-05
F_4^0	1.087***	1.387E-05	0.083*	0.044	0.024***	1.943E-12	0.173***	7.601E-08	1.258E-10	3.650E-06
F_5^0	0.356***	2.937E-04	0.122***	0.025	1.055***	1.532E-17	2.785***	3.974E-07	0.208***	1.823E-04
F_6^0	0.094***	2.917E-04	1.575***	0.001	0.443***	7.781E-12	0.262***	1.967E-05	0.182***	6.115E-06
A_1	-1.347***	0.001	-0.109***	0.004	9.150***	1.351E-08	0.903***	5.700E-05	0.988***	1.196E-04
A_2	-0.304***	0.002	0.192***	0.001	-9.059***	3.355E-09	0.287***	9.275E-06	-1.854***	2.533E-06
A_3	0.311***	0.001	0.069***	0.007	10.792***	2.295E-08	-5.974***	1.994E-04	-0.170***	1.740E-04
A_4	-1.158***	1.586E-04	-0.022***	0.006	13.517***	3.315E-08	-5.416***	2.231E-04	-0.380***	1.711E-04
A_5	0.769***	0.003	-0.161***	0.002	5.152***	3.421E-09	-0.807***	5.963E-05	3.298***	1.936E-05
A_6	-0.829***	0.003	-0.001	0.009	-26.060***	1.673E-08	0.227***	1.471E-05	-1.725***	2.740E-04
c	0.276***	0.005	-6.240***	0.003	-33.330***	2.784E-08	-16.146***	1.111E-05	-6.743***	0.001
C^0	-6.209***	1.320E-07	-5.910***	2.622E-04	-3.436***	1.382E-11	-3.510***	1.273E-06	-0.853***	1.469E-05
γ_1	-0.465***	2.373E-05	-0.688***	2.995E-04	-7.426***	1.908E-08	1.514***	3.238E-05	0.205***	1.520E-04
γ_2	0.027***	5.353E-06	-0.401***	0.001	1.609***	1.889E-08	0.244***	1.031E-05	-0.004***	2.853E-04
γ_3	-0.795***	5.490E-06	-2.446***	1.888E-04	11.006***	2.250E-08	5.557***	2.143E-04	-1.158***	2.620E-05
γ_4	-1.341***	2.041E-05	1.656***	5.918E-05	15.875***	2.818E-08	-6.218***	1.943E-04	-1.112***	5.844E-05
γ_5	-0.666***	1.355E-05	0.177***	4.425E-04	1.641***	1.074E-08	-1.635***	2.894E-05	-0.353***	0.001
γ_6	-1.353***	1.461E-05	-3.239***	3.641E-06	8.000***	5.433E-08	-0.710***	8.150E-06	1.779***	2.654E-04

Table 5: DAR: Results of the no-arbitrage test

Cryptocurrency	Ticker	$\hat{W}_T$	Critical Value*	Test Result
Binance Coin	BNB	1.165	3.841	*Cannot reject the null*
Bitcoin	BTC	7.573	3.841	*Reject the null*
Ethereum	ETH	4.565	3.841	*Reject the null*
USD Coin	USDC	1.473	3.841	*Cannot reject the null*
Tether	USDT	2.520	3.841	*Cannot reject the null*

* The value corresponds to the critical value for 95% confidence level.

4.2.2 Local Analysis of Individual Cryptocurrency Markets

In this section, we perform the analysis locally based on a rolling window approach with the window size equal to 100 days. More specifically, the SMM and the no-arbitrage test are applied locally to each cryptocurrency using only the observations from the last 100 days, with estimation sample ending at $T-200, T-199, \ldots, T-1, T$.

To summarize the results, for each cryptocurrency we calculate the percentage of days with no arbitrage opportunities, which is the total number of days for which we do not reject the null hypothesis of no-arbitrage as a percentage of the total number of estimation days, and report it in the last column of Table A.3 (for the standard AR model) in Appendix A and Table 6 (for the DAR model). We consider the results for the DAR model in Table 6 as our official test results and present the results in Table A.3 for an interesting comparison. Even if the test results in Table A.3 are technically less reliable (for the reason discussed in the global analysis), these results are similar to those in Table 6 for all five cryptocurrencies.

The tables show that, for Binance Coin, Bitcoin, and Ethereum, the percentage of days with no arbitrage opportunities varies between 59% and 85%. Therefore, although these cryptocurrencies do not seem to allow arbitrage opportunities on the majority of the days, they do appear to present investors with such opportunities frequently. On the other hand, there are no days with arbitrage opportunities for USD Coin and a very limited number of such days for Tether. As noted above in the global analysis, the absence or limited number of arbitrage opportunities in USD Coin and Tether could be explained by the fact that these cryptocurrencies are stablecoins. These results for USD Coin and Tether could also imply that these cryptocurrencies are locally efficient, which is in line with the findings of

Djogbenou, Inan, Jasiak (2023).

Table 6: DAR: Results of no-arbitrage test conducted locally

Cryptocurrency	Ticker	Total Est. Days	% of Days with No-arbitrage*
Binance Coin	BNB	200	78.50%
Bitcoin	BTC	200	60.50%
Ethereum	ETH	200	74.00%
USD Coin	USDC	200	100.00%
Tether	USDT	200	94.50%

* Percentage of days with no-arbitrage= 100×(total number of days with no arbitrage opportunity)/(total number of estimation days)

5 Conclusion

This paper contributes to the literature on the efficiency of the cryptocurrency markets by considering a general model for cryptocurrencies traded on multiple exchange platforms. The model assumes that the return of each cryptoasset varies in terms of a common fundamental risk factor and a specific idiosyncratic risk factor. The fundamental risk factor is modeled as a univariate Wishart autoregressive process (autoregressive gamma process), while the vector of the idiosyncratic risk factors is assumed to follow a conditional multivariate Gaussian distribution.

We provide an algorithm on how the simulation method of moments could be implemented to facilitate the estimation of the parameters in the model and conduct inferences. To assess the efficiency of cryptocurrency markets, we test the moment conditions implied by the no-arbitrage hypothesis and describe the procedure for its implementation.

The empirical results based on a global analysis show that fundamental risk plays an important role in explaining the dynamics of cryptocurrency returns and suggest the cryptocurrency markets are generally inefficient. A local analysis, where parameters are estimated sequentially using a rolling window, allows to mitigate those findings.

While the native cryptocurrency markets offer more frequent arbitrage opportunities, this inefficiency is drastically reduced in stablecoin markets. The analysis in this paper could be extended by modeling fundamental risk as a matrix that follows a Wishart autoregressive process and by considering alternative risk factors.

Appendix A: Standard AR Estimation Results

Table A.1: Standard AR: Global Parameter Estimates

Model Parameters	Binance Coin (BNB)		Bitcoin (BTC)		Ethereum (ETH)		USD Coin (USDC)		Tether (USDT)	
	Point Estimate	***Standard Deviation***	***Point Estimate***	***Standard Deviation***	***Point Estimate***	***Standard Deviation***	***Point Estimate***	***Standard Deviation***	***Point Estimate***	***Standard Deviation***
μ	-0.998	2.133	0.996	1.603	0.999	3.204	1.000***	0.249	0.098***	5.167E-20
Ω	0.052	0.148	0.327	0.276	0.012	0.295	0.034**	0.014	0.046***	4.298E-18
f_1	-3.931***	0.021	0.166	2.088	0.074	4.048	-1.484***	0.012	0.272***	5.566E-17
f_2	-0.213	0.404	-2.764***	0.032	0.189	20.142	0.344	0.494	1.639***	2.992E-17
f_3	-1.509***	0.029	0.162	1.076	0.091	45.232	0.208	0.175	0.247***	7.875E-18
f_4	1.564**	0.701	0.226	2.191	0.143	27.864	0.346	0.389	-0.417***	3.474E-19
f_5	-0.070	0.963	0.322	1.735	0.037	58.394	-0.249	0.219	-0.220***	2.027E-17
f_6	-0.629**	0.254	0.054	0.070	0.003	52.744	-0.218	0.271	-0.102***	2.574E-18
F_1^0	4.644***	0.001	0.762***	0.272	0.659***	0.061	1.605***	0.001	0.004***	6.027E-21
F_2^0	0.885***	0.011	0.789***	0.001	0.006	0.299	0.128***	0.049	0.002***	3.292E-21
F_3^0	0.036***	0.001	0.040	0.144	0.582	0.680	0.092***	0.017	0.007***	9.071E-22
F_4^0	1.123E-08	0.015	0.807***	0.280	0.329	0.417	0.525***	0.041	0.004***	4.551E-23
F_5^0	1.255E-08	0.019	0.230	0.219	0.790	0.883	0.486***	0.024	0.162***	2.206E-21
F_6^0	0.849***	0.006	0.377***	0.010	0.905	0.800	0.440***	0.029	0.185***	2.988E-22
A_1	1.063***	0.047	-0.122	0.185	-0.140***	0.022	0.197***	0.006	7.738***	2.631E-20
A_2	1.010***	0.005	-0.240***	0.007	-0.694***	0.099	-0.522***	0.004	-3.513***	1.616E-21
A_3	-0.307***	0.035	-0.202**	0.091	-0.320*	0.194	-0.058***	0.003	3.889***	5.774E-21
A_4	0.090	0.192	-0.133	0.177	-0.182	0.119	0.107***	0.007	1.540***	1.182E-20
A_5	-2.407***	0.013	-1.072***	0.144	-0.046	0.280	0.183***	0.006	-1.343***	6.985E-21
A_6	-0.013	0.009	0.011	0.034	-0.002	0.259	0.476***	0.010	-1.237***	3.068E-20
c	-2.444***	0.562	-0.376	0.386	-0.002	3.923	-0.312***	0.096	-39.562***	2.708E-17
C^0	-17.371***	0.012	-7.689***	0.028	-3.173***	0.056	-2.408***	0.010	-3.039***	2.923E-21
γ_1	1.529***	0.033	-1.034***	0.005	-0.487***	0.004	-0.780***	0.002	-0.284***	9.117E-20
γ_2	0.060*	0.031	0.174***	0.009	0.073***	0.020	-0.057***	0.005	-0.024***	4.139E-20
γ_3	1.097***	0.010	0.139***	0.007	-0.652***	0.009	-0.195***	0.001	0.406***	4.583E-20
γ_4	5.551***	0.003	-1.703***	0.005	-1.105***	0.005	-1.457***	0.001	1.002***	1.815E-20
γ_5	-0.008	0.075	-0.225***	0.040	-0.584***	0.001	-0.595***	0.002	0.272***	1.582E-20
γ_6	-0.235***	4.118E-04	-0.891***	4.185E-04	-2.421***	4.646E-05	1.035***	0.004	2.607***	1.457E-20

Table A.2: Standard AR: Results of the no-arbitrage test

Cryptocurrency	**Ticker**	$\hat{W}_T$	**Critical Value***	**Test Result**
Binance Coin	BNB	267.268	5.991	*Reject the null*
Bitcoin	BTC	32098.664	5.991	*Reject the null*
Ethereum	ETH	392.598	5.991	*Reject the null*
USD Coin	USDC	4.070	5.991	*Cannot reject the null*
Tether	USDT	2.742	5.991	*Cannot reject the null*

* The value corresponds to the critical value for 95% confidence level.

Table A.3: Standard AR: Results of no-arbitrage test conducted locally

Cryptocurrency	**Ticker**	**Total Est. Days**	**% of Days with No-arbitrage***
Binance Coin	BNB	200	85%
Bitcoin	BTC	200	59%
Ethereum	ETH	200	60%
USD Coin	USDC	200	100%
Tether	USDT	200	96%

* Percentage of days with no-arbitrage= 100×(total number of days with no arbitrage opportunity)/(total number of estimation days)

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