Estimation of Credit Rating Transition Under Stress Scenarios

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Abstract

This paper estimates the credit rating transitions from a stochastic factor Probit model and examines the effects of one time shocks to the economy on credit rating and default probability. The estimation method considered maximizes the analytical approximations of the log-likelihood function based on the granularity theory. The stochastic factor Probit model includes a common latent factor that reveals the effect of the states of the economy on corporate credit ratings under stress scenarios. The unobserved factor, future credit ratings, and corporate default probabilities are subject to a stress test of a one time shock applied to the factor. The shocks are used to evaluate stressed migration probabilities and default probabilities during and after the crisis of 2008. Moreover, the shock effects help us determine without ambiguity which macro variable has factor characteristics. The model is estimated from annual data on transition matrices provided by the internal rating system of a European Bank over the period of 2007 through 2014.

Keywords : Approximate Likelihood, Stochastic Factor Ordered Probit, Stress Testing.

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1 Introduction

In the financial industry, finding appropriate methodologies to control and regulate risks has always been in the center of attention, especially after the last financial crisis of 2008. Different techniques have been developed in the literature, such as simulation-based methods for computing the risk measures and estimating the model parameters. However, they can be very time consuming and also can be insufficient in some cases like, computation of capital reserves or investigating the effects of model parameters and risk factors on portfolio risk. For these purposes, analytical approximations of the risk measures become of interest. An approach that relaxes the assumption of infinity large portfolios and provides the approximated value of a risk measure for a large but not infinity large portfolios is the so-called granularity principle. The Basel Committee on Banking Supervision (BCBS) suggests banks use this methodology for their credit risk analysis and their computation of capital reserves in an early draft of Basel II. [see, e.g. Basel Committee on Banking Supervision (2001); Wilde (2001); Martin and Wilde (2002); Gordy (2003, 2004); Gagliardini et al. (2012); Gordy and Lütkebohmert (2007); Gagliardini and Gouriéroux (2013)].

The objective of the paper is to estimate a stochastic credit migration Probit model with a serially correlated factor and study by simulations the effect of shocks to the factor on migration probabilities. The stochastic migration model is a nonlinear dynamic panel model with a common unobserved factor. Such a model is often encountered in credit risk studies and used to analyze the dynamics of corporate ratings. Moreover, it is used for the prediction of future credit risk in a homogeneous pool of credits [see, e.g. Gupton et al. (1997); Gordy and Heitfield (2002); Gagliardini and Gouriéroux (2005); Feng et al. (2008)]. The ordered qualitative model with one factor is a basic stochastic migration model and it is an extension of the Asymptotic Single Risk Factor (ASRF) model as well. We examine the effects of one-time shocks to the factor on credit migrations and default probabilities. Our analysis reveals that the shock effects help determine without ambiguity which macroeconomic variable has the same characteristics as the factor and can be interpreted as such. This finding has implications for policy making during economic downturns, such as crisis of 2008 or Covid-19 epidemics. The true log-likelihood function of the stochastic migration model involves a large dimensional integral due to the presence of latent factors that need to be integrated out. Therefore, the computation of the true log-likelihood is complicated, see, e.g. Schönbucher (2001, 2003); Frey and McNeil (2003) for a numerical implementation of the model.

The stochastic migration model is estimated by the two-step estimation method which is the simplified maximum likelihood based on the granularity theory, providing asymptotically efficient estimators [see, Gagliardini and Gourieroux (2014)]. The parameter set is divided into micro-parameters that determine the credit ratings and macro-parameters that define the factor dynamics. In the first step, the factor values are considered as nuisance parameters and the estimator works approximately as a fixed effects estimator. In the second step, the unobserved factor values are replaced by cross-sectional factor approximations and the estimator of the micro-parameters is obtained.

The stress testing analysis is applied to internal credit rating of a large European Bank. It allows us to interpret the factor, which points to the short-term interest rate. We observe a noticeable decrease in the total default rates of all rating categories in the year we introduce a one-time positive shock to the factor and increase in the total default rates of all rating categories when we introduce a one-time negative shock to the factor. We examine the effect of one-time shocks to the factor on the probabilities of transitions between credit ratings. The stress scenarios are applied to the economy during the downturn of 2008 and after the recovery in 2010. We observe that the shock effects differ and are more apparent in the post-crisis economy. During the financial crisis in 2008, the short term interest rate in France decreases by 0.32%, however, our finding, shows that this magnitude of the decrease is not quite significant to prevent the rise in the downgrade rates and default rates of firms in France. Our findings show that a shock either positive or negative with the size of one standard deviation of the estimated factor (0.56) has a significant impact on the firm's downgrade rates and default probabilities in France. The size of the shock is about the one third of the standard deviation of the short-term interest rate in France, which is 1.71 in our sample period. Our findings suggest to lower the short-term interest rate to improve the credit ratings and default probabilities of firms in times of of economic difficulties such as the financial crisis in 2008 or the COVID-19 pandemic in 2020. Moreover, our finding shows that the migration probabilities to and from category "B" are those that are mostly affected. The downgrade rate of firms rated "B" decreases by 0.18% when we introduce the positive shock and increases by 0.22% when we introduce the negative shock to the factor. The one-time positive shock causes a decrease in the total default rates of all rating classes by 0.61% and the one-time negative shock increases the total default rates of all classes by 0.68%.

The paper is organized as follows. Section 2 presents the stochastic migration model. Section 3 discusses two estimation methods. First, an approximate linear state-space model as the benchmark model and second, the two-steps efficient estimation methodology of the stochastic factor Probit model are considered. Section 4 presents the description of the dataset. The results of these two estimation approaches are discussed in Section 5. In Section 6 we compare the estimated and observed migration matrices. We perform stress testing analysis in Section 7. Section 8 concludes the paper. Appendices A-E contain additional tables.

2 Stochastic Migration Model

The effect of current ratings of borrowers on the value of their debts in a credit portfolio of a financial institution has to be taken into account when the risk of the most credit portfolio is assessed. An accurate analysis of the risk associated with possible rating downgrades and upgrades is a requirement of the Basel 2 accord. [Basel Committee on Banking Supervision (2001, 2003)] Thus, it is mandatory to perform dynamic analysis of qualitative rating histories that are provided by the rating agencies, such as the Moody's or large banks. Let $y_{i,t}$ denote the rating of firm $(i), i = 1, ..., n$ at time $t = 1, ..., T$. The ratings are denoted by $k = 1, ..., K$ and define the credit quality of firm (i) at time t:

$$y_{i,t} = k, \quad k = 1, ..., K$$

In order to model the dynamics of a large number of qualitative individual histories $(y_{i,t})$, $i = 1, ..., n, \;\; t = 1, ..., T$ over the finite state $(1, ..., K)$, we use the stochastic transition model. Individual firm (i) has a credit rating history which is denoted by $(y_{i,t}, \;\; t = 1, ..., T)$, $i = 1, ..., n$ and corresponds to the qualitative process with state-space $(1, ..., K)$ at any point of time t. These states indicate all possible rating categories such as $A^+, A, ..., F$ of firm $i = 1, ..., n$ and characterize different risk categories. State (A^+) refers to the low-risk category and (F) refers to state default, which is an absorbing state and the highest risk category.

The object of our analysis is a sequence of square matrices $\hat{P}_t$ with positive elements, which are the frequencies of transitions between states. Each element of these matrices refers to frequencies of transitions from one rating category to another in a unit of time. The elements of each matrix are positive and each row sums up to one since they approximate the probabilities of migration. The dimensions of each matrix depend on the number of states K. The matrices are updated and reported each year by the rating agencies such as the Moody's or by large banks. The analysis of such matrices can be performed either under the assumption that matrices of transition probabilities $P_t, \;\; t = 1, ..., T$ are time-independent (time homogeneity assumption) or time-varying (heterogeneity assumption). We consider stochastic time-varying matrices in our analysis and introduce two main assumptions:

Assumption 2.1- The individual rating histories ($y_{i,t}\ \ t = 1, ..., T$), are i.i.d. for $i = 1, ..., N$ and follow a heterogeneous Markov process of state 1 with transition matrices $\{P_t\}$.

Assumption 2.2- The sequence $\{P_t\}$ is stochastic.

The heterogeneous Markov process means that all the information on the rating history of a given firm can be explained by the most recent individual rating and that the migration probabilities are time-varying. The transition probabilities are the same for different individuals, which follows from the cross-sectional homogeneity assumption on the population of firms.[1]

Based on assumptions 2.1 and 2.2 the joint dynamics of rating histories of all firms are characterized by a sequence of migration matrices $P_t,\ t = 1, ..., T$. The elements of migration matrices describe the probability of movement of each firm ratings from one rating category to another. The dimension of the transition matrices is $(K \times K)$ and each entry $p_{l,k,t}$ displays the probability of changing in firm's rating from (l) to (k) between time $(t-1)$ and (t):

$$p_{l,k,t} = P[y_{i,t} = k | y_{i,t-1} = l, f_t],\ \forall k, l, t, \tag{2.1}$$

where $f_t, t = 1, ..., T$ is a common unobserved factor.

The qualitative ratings $y_{i,t}$ depend on an unobserved continuously valued score $y^*_{i,t}$. By discretizing the score grade, the qualitative rating of corporates can be defined, as follows: The quantitative values of score y^*_{it} can be between $c_0 = -\infty < c_1 \leq ... \leq c_K = \infty$. Based on the score value, the ratings are defined by:

$$y_{i,t} = k, \quad \text{iff} \quad c_{k-1} \leq y^*_{i,t} < c_k, \quad k = 1, ..., K. \tag{2.2}$$

The rating classes are numbered in order of increasing credit quality, with alternative $k = 1$ corresponding to the highest risk category or default (F) and $k = K$ corresponding to the lowest risk category ("A+"). The movements of firms from one rating category to another depends on a common unobserved factor f_t. Therefore, it is important to specify the conditional distribution of quantitative score given the factor and the past individual histories:

$$y^*_{i,t} = \delta_l + \beta_l f_t + \sigma_l u_{i,t}, \quad \text{if} \quad y_{i,t-1} = l,\ l = 1, ..., K. \tag{2.3}$$

where $u_{i,t} \sim IIN(0, 1)$ is independent of f_t. By conditioning, with respect to the last rating, we obtain a set of K homogeneous sub-populations at date t. It means that firms are grouped into rating categories $k = 1, ..., k$ according to their previous rating. Furthermore, the unobserved common factor satisfies a Gaussian Autoregressive (AR(1)) model:

$$f_t = \mu + \rho f_{t-1} + \eta_t, \qquad \eta_t \in IIN(0, \sigma^2_\eta),\ t = 1, ..., T \tag{2.4}$$

and the errors $u_{i,t}, \eta_t$ are identically distributed and independent. We assume of $\rho \in (-1, 1)$, to ensure that stationary behavior of the factor. Thus, our dynamic model is defined by a state and measurement equations as follows:

State equation: It is defined by the conditional density of the factor: $\psi(f_t | f_{t-1}; \rho), \forall t$

Measurement equations: These equations are defined by the transition probabilities:

$$\begin{aligned} p_{lk,t}(f_t; \theta) =& P[y_{i,t} = k | y_{i,t-1} = l, f_t; \theta] \\ =& P[c_{k-1} \leq y^*_{it} < c_k | y_{it-1} = l], \end{aligned} \tag{2.5}$$

for $l, k = 1, ..., K$ and θ, a vector of parameters of the model given below.

[1] See, Gagliardini and Gourieroux (2014), Definition 1.1, Page 6.

The transition density function of $y_{i,t}$, given $y_{i,t-1}$ and f_t is characterized by the $(K \times K)$ migration matrix, since the individual observations are qualitative. Under (2.2), (2.3) and (2.5) the dynamic model is a Probit with the transition probabilities given by:

$$p_{lk,t}(f_t;\theta) = \Phi(\frac{c_k - \delta_l - \beta_l f_t}{\sigma_l}) - \Phi(\frac{c_{k-1} - \delta_l - \beta_l f_t}{\sigma_l}),\ l,k = 1,...,K,\ t = 1,...,T \qquad (2.6)$$

where Φ is the standard normal cumulative distribution function (CDF) in this framework [2]. There are two kinds of parameters: the micro-parameters and macro-parameters, which are the parameter vector θ including the thresholds c_k,
$k = 1,...,K-1$ and parameters of latent score function, $(\delta_l, \beta_l, \sigma_l),\ l = 1,...,K$, and parameter ρ characterizes the factor dynamics, respectively.

The vector of micro-parameters θ characterize the micro-dynamics of ratings and macro-parameter ρ characterizes the factor dynamic. The micro-dynamics are defined by the measurement equation and the macro-dynamics are defined by the state equation and both are conditional on a given factor path. Moreover, the individual histories of the credit ratings of all firms are influenced by a common factor f_t and this common factor creates cross-sectional dependence between individual histories. Since the factor is unobservable, it has to be integrated out. Therefore, the true log-likelihood function is given by:

$$log\ l(\underline{y_T};\theta,\rho),$$

where:

$$l(\underline{y_T};\theta,\rho) = \int ... \int \left([\prod_{t=1}^{T} \psi(f_t|f_{t-1};\rho)] \prod_{t=1}^{T}\prod_{i=1}^{n} f(y_{i,t}|y_{i,t-1},f_t;\theta) \prod_{t=1}^{T} df_t \right), \qquad (2.7)$$

The log-likelihood function involves multiple integrals of a large dimension equal to the time dimension T. The dimension of this integral increases with T. Generally, n refers to the number of companies and the time dimension T depends on the frequency of the observations, which can be yearly or monthly. There exist three different settings for asymptotic analysis: both n and T are large, n is large and T is fixed and, n is fixed and T is large. In the case of large n and T, asymptotically efficient estimators of parameters of the above log-likelihood are available without computing the high dimensional integral. In the model considered in this paper, n is large and T is fixed. Due to the complexity of the log-likelihood (2.7), we use the simplified likelihood estimation methodology based on the granularity theory. [see, Gagliardini and Gouriéroux (2010); Gagliardini and Gourieroux (2014)]

3 Estimation of the Stochastic Migration Model

In this section, we review the likelihood-based estimation methods for a stochastic factor Probit model. These methods are based on analytical approximations of the log-likelihood function that are derived from the so-called granularity principle. [see, Feng et al. (2008), Gagliardini and Gouriéroux (2010); Gagliardini and Gourieroux (2014)]

In Section (3.1), we rewrite the stochastic migration model as an approximate linear state space model. Next, the approximate linear state-space model is estimated by applying the standard Kalman filter which provides consistent and fully efficient estimators of the micro-parameters of the model and also the approximated factor path when the cross-sectional dimension n, the number of firms in the sample, is large. If the cross-sectional dimension n is not sufficiently large this method loses its optimality. However, we can still use the

[2]Each row of the transition matrix, conditional on (f_T) contains an ordered Probit model with a common factor (f_t). When factors serially correlated, as in (2.4), the transition matrices are serially correlated.

estimates of the parameters as initial values in the numerical algorithm to maximize the log-likelihood of the two-step method. An example of the approximate linear state-space model and estimation procedures is given in Appendix A to clarify the estimation procedure.

In Section (3.2), the stochastic factor Probit model is estimated by the two-step estimation method. In the first step, the factor values are considered as nuisance parameters and the estimator of micro-component arises as a fixed effects estimator. In the second step, the unobservable factor values are replaced by a cross-sectional factor approximation and the estimator of the macro parameter is obtained by maximizing the likelihood of the macro-dynamics for example by applying the maximum likelihood (ML) estimator to the autoregressive AR(1) model.

3.1 Approximate Linear State Space Model

The stochastic migration model can be represented as an approximate linear state space model. In this approach, for each row of the transition matrix, a canonical factor is computed. An estimation approach that relies on this linearized qualitative model is straight forward. Let us recall the equations (2.5) and (2.6) .

$$\begin{aligned} p_{lk}(f_t;\theta) &= P[y_{i,t} = k | y_{i,t-1} = l, f_t; \theta] \\ &= \Phi(\frac{c_k - \delta_l - \beta_l f_t}{\sigma_l}) - \Phi(\frac{c_{k-1} - \delta_l - \beta_l f_t}{\sigma_l}),\ k = 1, ..., K,\ t = 1, ..., T \end{aligned}$$

We deduce the cumulative transition probabilities p^*_{lk} for each row as follows:

$$\begin{aligned} p^*_{lk}(f_t;\theta) &= P[y_{i,t} \leq k | y_{i,t-1} = l, f_t; \theta], \\ &= \sum_{h=1}^{k} p_{lh}(f_t;\theta), \\ &= \Phi(\frac{c_k - \delta_l - \beta_l f_t}{\sigma_l}), \end{aligned} \tag{3.1}$$

for $l = 1, ..., K$ and $k = 1, ..., K-1$. If we apply the quantile function of the standard normal distribution to both sides of equation (3.1) we get:

$$\Phi^{-1}[p^*_{lk}(f_t;\theta)] = \frac{c_k - \delta_l - \beta_l}{\sigma_l} = \frac{c_k - \delta_l}{\sigma_l} - \frac{1}{\sigma_l}\beta_l f_t, \tag{3.2}$$

These nonlinear transformations of the cumulative transition probabilities for $l = 1, ..., K$ and $k = 1, ..., K-1$, play the role of the canonical factors which are linear with regard to f_t, and are denoted as:

$$a_t = vec[a_{l,k,t}], \tag{3.3}$$

where

$$a_{l,k,t} = \frac{c_k - \delta_1 - \beta_l f_t}{\sigma_l} = \frac{c_k - \delta_l}{\sigma_l} - \frac{1}{\sigma_l}\beta_l f_t, \tag{3.4}$$

The probability $p^*_{lk,t}$ is well approximated by its cross-sectional sample counterpart $p^*_{lk,t}$ when the cross-sectional dimension n is large ($n \to \infty$). Furthermore, the estimated canonical factors $\hat{a}_{l,k,t}$ are such that

$$\hat{a}_{l,k,t} = \Phi^{-1}\Big(\sum_{h=1}^{k} \hat{p}_{l,h,t}\Big), \tag{3.5}$$

Therefore, for a large cross-sectional dimension n, we get the estimates of canonical factors $\hat{a}_t$, $t = 1, ..., T$ which are asymptotically normally distributed with their estimated asymptotic variance-covariance matrix $\hat{\Sigma}_{n,t}$,

$$\hat{a}_t \overset{d}{\sim} N(a_t, \hat{\Sigma}_{n,t}),$$

The derivation of the asymptotic variance-covariance matrix $\hat{\Sigma}_{n,t}$ is given in [Gagliardini and Gourieroux (2014)]. The state equation and the measurement equation can be written as follows:

State equation:

$$f_t = \mu + \rho f_{t-1} + \eta_t, \quad \eta_t \sim IID(0,1), \; t = 1, ..., T,$$

Measurement equation:

$$\hat{a}_{l,k,t} = \frac{c_k - \delta_l}{\sigma_l} - \frac{\beta}{\sigma_l} f_t + u_{l,k,t}, \quad vec(u_{l,k,t}) \sim IIN(0, \hat{\Sigma}_{n,t}),$$

for $l = 1, ..., K$ and $k = 1, ..., K$.

In this approach, the nonlinear measurement equations (2.5) are approximated by the linear measurement equations for a sufficiently large cross-sectional dimension n and can be analyzed by the standard linear Kalman filter. The estimates of parameters $c, \beta, \delta,$ and σ can be obtained by the standard Kalman filter under the following identification restrictions:

First, due to an invertible affine transformation of the factor, the identifying constraints can be imposed on the factor dynamics:

$$E(f_t) = 0, \text{ and } V(f_t) = 1,$$

Second, since, the quantitative score is partially observable, we need to impose the standard identification restrictions for an ordered Probit model for one row only. The identification restrictions for micro-parameters in this model concern the parameters of rating class ('B'):

$$\delta_4 = 0 \text{ and } \sigma_4 = 1,$$

After imposing these 4 restrictions, and given $E(f_t) = 0$, all remaining parameters can be obtained from:

$$\frac{c_k - \alpha_l}{\sigma_l} = \frac{1}{T} \sum_{t=1}^{T} \hat{a}_{l,k,t},$$

$$\frac{\beta_l}{\sigma_l} = \text{Standard error of } \; \hat{a}_{l,k,t},$$

for $l = 1, ..., K$ and $k = 1, ..., K-1$.

If the cross-sectional dimension n tends to infinity, the errors in the measurement equations tends to zero. Therefore, we get an approximated linear state space model in which the macro-component is determined from the transition equation and the micro-components from the measurement equation. The standard linear Kalman filter can be applied to estimate this approximated model under the identification restrictions. In this approach, the rate of convergence for the micro-parameters is $\frac{1}{\sqrt{nT}}$ and for the macro-parameter is $\frac{1}{\sqrt{T}}$. This approach provides estimates of the micro-parameters, c, δ, β, σ and also allow us filter the latent factor values. The factors can be obtained by spectral decomposition of the $(T \times T)$ matrix YY', where the row t of matrix Y is given by $(\hat{a}_{l,k,t} - \bar{a}_{l,k})$, where, $(\bar{a}_{l,k} = \frac{1}{T} \sum_{t=1}^{T} a_{l,k,t})$. We provide an example of the model with three rating categories $k = 3$ in Appendix A.

3.2 The Two-step Efficient Estimation Approach

Under the granularity theory, the approach is the following. The joint density of variables $y_{i,t}, \; i = 1, ..., n, \; t = 1, ..., T$ and of factor $f_t, \; t = 1, ..., T$ (conditional on the initial observations) if both individual ratings $y_{i,t}$ and f_t were observed would be as follows as follows:

$$l^*(\underline{y_T}, \underline{f_T}; \theta, \rho) = [\prod_{t=1}^{T} \psi(f_t | f_{t-1}; \rho)] \prod_{t=1}^{T} \prod_{i=1}^{n} f(y_{i,t} | y_{i,t-1}, f_t; \theta),$$

The log-likelihood function can be defined and decomposed as:

$$\mathcal{L}^*(\underline{y_T}, \underline{f_T}; \theta, \rho) = log\ l^*(\underline{y_T}, \underline{f_T}; \theta, \rho) = L^M(f_T; \rho) + \sum_{t=1}^{T} L^{CS}(\tilde{y}_t|\tilde{y}_{t-1}, f_t; \theta),$$

The first component $L^M(f_T; \rho)$ corresponds to the macro-log-likelihood function and the second component, $\sum_{t=1}^{T} L^{CS}(\tilde{y}_t|\tilde{y}_{t-1}, f_t; \theta)$ corresponds to the cross-sectional micro log-likelihood function:

Macro log-likelihood:

$$L^M(f_T; \rho) = \sum_{t=1}^{T} \log\ \psi(f_t|f_{t-1}; \rho), \tag{3.6}$$

Micro log-likelihood:

$$L^{CS}(\tilde{y}_t|\tilde{y}_{t-1}, f_t; \theta) = \sum_{i=1}^{n} \log\ f(y_{i,t}|y_{i,t-1}, f_t; \theta), \tag{3.7}$$

As already mentioned, the true log-likelihood function (2.7) is too complex. Therefore, the $l(\underline{y_T}; \theta, \rho)$ is replaced by $\mathcal{L}^*(\underline{y_T}, \underline{f_T}; \theta, \rho)$.

If the micro-parameters θ are known, the factor value at time t can be approximated by a fixed effects estimator. Thus, in the first step we get the factor values at time t as follows:

- Step 1:

$$\hat{f}_{n,t}(\theta) = \arg\max_{f_t} \sum_{i=1}^{n} logf(y_{i,t}|y_{i,t-1}, f_t; \theta), \tag{3.8}$$

In this approach $\hat{f}_t(\theta)$ is treated as a parameter in the latent cross-sectional micro-likelihood, and the value of the factor is estimated as a fixed time effect. In the second step, the factor approximations $\hat{f}_n, t(\theta)$ are reintroduced in the latent micro-likelihood functions, aggregated over time, to obtain a function of the observations $\underline{y_T}$ and parameter θ estimated follows:

- Step 2:

$$\hat{\theta}_{n,T} = \arg\max_{\theta} \sum_{t=1}^{T} \sum_{i=1}^{n} logf(y_{i,t}|y_{i,t-1}, \hat{f}_{n,t}(\theta); \theta), \tag{3.9}$$

Next, an approximation of factor value at time t can be defined by reintroducing the estimated micro-parameters in the expression of the fixed effects estimator of the factor values as follows:

$$\hat{f}_{n,T,t} = \hat{f}_{n,t}(\hat{\theta}_{n,T}) \tag{3.10}$$

Next, after estimating T values of factors, the estimator of the macro-parameter ρ can be simply the Maximum Likelihood (ML) estimator of the autoregressive model as follows:

$$\hat{f}_{n,T,t} = \mu + \rho \hat{f}_{n,T,t-1} + \eta_t, \quad \eta_t \sim IIN(0, \sigma_\eta^2), \quad t = 1, ..., T. \tag{3.11}$$

In the first step, we use the estimates of micro-parameters of the approximate linear model. Thus, the micro-parameters can be considered as known parameters. Then, we estimate our model in two-step as explained above. For the two-step estimation method it is enough to impose the identifying restrictions for partial observability only ($c_3 = \delta_4 = 0$ and , $\beta_4 = \sigma_4 = 1$). Since the cross-sectional dimension in our framework is much larger than the time dimension, the unknown factor values can be treated as nuisance parameters without having an incidental parameter problem. This problem arises when the time dimension T tends to infinity. [see, Neyman et al. (1948), Gagliardini and Gourieroux (2014)].

Gagliardini and Gourieroux (2014) derive the asymptotic properties of the micro- and macro-parameter. The estimators converge with different rates of convergence to their corresponding true values when both n and T tend to infinity. The micro-parameters $\hat{\theta}_{nT}$ are $\sqrt{nT}$ consistent and the rate of convergence is $\frac{1}{\sqrt{nT}}$. The macro-parameters $\hat{\rho}_T$ is $\sqrt{T}$ consistent and the rate of convergence is $\frac{1}{\sqrt{T}}$. The factor estimates $\hat{f}_t$ are $\sqrt{n}$ consistent and the rate of convergence is $\frac{1}{\sqrt{n}}$. The parameters are asymptotically efficient and they are asymptotically equivalent to the maximum likelihood estimators. In the case where both n and T are large, the lowest possible variance can be obtained for the estimators $\hat{\theta}_{n,T}$ and $\hat{\rho}_T$. [see, Gagliardini and Gourieroux (2014) for the further discussion on the asymptotic properties of the micro- and macro parameters]

4 Description of the Data

The data set consists of 8 (eight) migration matrices that contain the internal rating data provided by a European bank in France. These matrices were recorded annually from 2007 to 2015, hence $T = 8$ in our sample. There are 14 risk categories in that European bank credit rating system. These categories are "1", "2", "3", "4", "5", "6", "7", "8", "9", "10", "11", "12", "13" and "14", from the lowest to the highest risk. All rating movements over a one year period are reported in transition matrices. These movements are migration from one rating to another, except for the default category which is the absorbing state and once a company joins the default category, there is no exit from it. Table 1 shows an example of a transition matrix for the year 2007, which has 14 categories of risks. The rating categories from which transitions are to be made are represented in rows. The first row refers to the rating category "1", and the last row shows the category "13". However, the category "14", is excluded from the rows, since no company can exit from the default category. Columns 1 to 14 correspond to rating categories "1" to "14" to which transitions will be made until the end of the year 2007. The numbers of long-term rated issuers per rating category at the beginning of the year 2007 are reported in the issuer column. Column 15, "NR", relates to issuers, who were rated at the beginning of the year 2007 but were not rated at the end of the year 2007. From the statistical point of view, there are reasons that cause this lack of information. For example, sometimes ratings cannot be assigned to a firm. For instance, when the debt of a firm is completely paid off or when a firm is terminated and the relevant debt extinguished, the rating of that firm is eliminated [see Brady et al. (2002)]. Each element in Table 1 represents the observed transition frequencies for the year 2007. For instance, the second row shows that, out of "322' firms rated "2" at the beginning of the year 2007, 2.8 % was upgraded to "1", 27.33% stayed in the same rating category, and 24.53% were downgraded to rating category "3". Typically, the highest percentages are found on the main diagonal of the transition matrix, because most of the firms remain in the same credit ratings category. The last column represents the proportion of non-rated corporations.

In the literature, the following two alternative approaches were considered, which are to either include the information on transition probabilities of non-rated firms at the beginning of the year, or to exclude the information on transition probabilities of non-rated firms at the end of the year. The first approach requires information on companies that are not rated at the beginning of each year. However, since this information is not easily available, the second option is often considered. Nickell et al. (2000), Bangia et al. (2002) and Foulcher et al. (2005), have all excluded the information on "NR" firms from the transition matrices and they allocated the weights of "NR" firms among the other risk categories. In this paper, the same approach is followed. Table 2 shows the matrix of migration probabilities from

Table 1: Number of Issuers and Transition Matrix for Year 2007 in %

2007	Issuers	1	2	3	4	5	6	7	8	9	10	11	12	13	14	NR
1	91	53	13	13	10	3	2	4	0	0	0	1	0	0	0	1
2	322	2.8	27.33	24.53	11.8	7.76	9.63	3.42	5.59	3.42	1.24	0.31	1.24	0.62	0.31	0
3	1132	0.88	2.39	28.62	11.93	13.34	16.43	9.36	6.45	4.51	3.36	1.41	0.62	0.62	0.09	0
4	3181	0.03	0.16	2.77	23.45	14.4	19.96	15.72	11.76	5.94	2.92	1.54	0.53	0.5	0.31	0.01
5	2904	0.07	0.03	2.07	4.75	34.92	13.5	15.5	12.36	8.54	3.99	2.1	0.93	0.55	0.69	0
6	8120	0.01	0.1	0.83	1.98	7.46	29.31	17.4	19.47	12.4	5.99	2.46	0.92	0.84	0.83	0
7	7263	0.04	0.1	0.54	0.81	3.95	6.83	36.09	18.24	18.08	9.14	3.37	1.25	0.77	0.78	0.01
8	6252	0.03	0.03	0.22	0.75	2.91	5.52	11.72	42.24	17.93	11.28	3.82	1.6	0.86	1.07	0.02
9	14256	0.02	0.04	0.14	0.32	1.36	3.31	9.48	13.73	38.53	20.19	6.76	2.62	1.98	1.52	0
10	9207	0.01	0.02	0.09	0.24	1.08	1.8	6.1	9.51	15.9	43.04	10.49	4.95	3.65	3.11	0.01
11	3356	0.03	0	0.12	0.06	0.6	1.4	3.81	5.9	11.86	19.85	34.51	9.71	6.94	5.21	0
12	2641	0.04	0	0.08	0.38	0.27	1.1	3.1	5.26	9.01	16.74	13.86	31.58	11.66	6.93	0
13	2451	0.08	0.12	0	0.16	0.69	1.43	1.55	3.59	6	9.63	9.47	9.71	48.92	8.65	0

Table 1 adjusted for N.R. For instance, in the first row of the "N.R.-adjusted" transition matrix the frequency of transition from "1" to "1" is 53.53% which is calculated from the ratio $53/(1-0.01)$.

Table 2: Adjusted Transition Matrix for Year 2007

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	53.54	13.13	13.13	10.10	3.03	2.02	4.04	0.00	0.00	0.00	1.01	0.00	0.00	0.00
2	2.80	27.33	24.53	11.80	7.76	9.63	3.42	5.59	3.42	1.24	0.31	1.24	0.62	0.31
3	0.88	2.39	28.62	11.93	13.34	16.43	9.36	6.45	4.51	3.36	1.41	0.62	0.62	0.09
4	0.03	0.16	2.77	23.45	14.40	19.96	15.72	11.76	5.94	2.92	1.54	0.53	0.50	0.31
5	0.07	0.03	2.07	4.75	34.92	13.50	15.50	12.36	8.54	3.99	2.10	0.93	0.55	0.69
6	0.01	0.10	0.83	1.98	7.46	29.31	17.40	19.47	12.40	5.99	2.46	0.92	0.84	0.83
7	0.04	0.10	0.54	0.81	3.95	6.83	36.09	18.24	18.08	9.14	3.37	1.25	0.77	0.78
8	0.03	0.03	0.22	0.75	2.91	5.52	11.72	42.25	17.93	11.28	3.82	1.60	0.86	1.07
9	0.02	0.04	0.14	0.32	1.36	3.31	9.48	13.73	38.53	20.19	6.76	2.62	1.98	1.52
10	0.01	0.02	0.09	0.24	1.08	1.80	6.10	9.51	15.90	43.04	10.49	4.95	3.65	3.11
11	0.03	0.00	0.12	0.06	0.60	1.40	3.81	5.90	11.86	19.85	34.51	9.71	6.94	5.21
12	0.04	0.00	0.08	0.38	0.27	1.10	3.10	5.26	9.01	16.74	13.86	31.58	11.66	6.93
13	0.08	0.12	0.00	0.16	0.69	1.43	1.55	3.59	6.00	9.63	9.47	9.71	48.92	8.65

In addition, for computational simplicity, the rating categories are reduced from 14 to 7 categories. In this paper, for computational simplicity, these 14 categories are reduced to 7 categories as follows. We keep the two credit ratings "1" and "2" that are considered high grades or investment groups and the two categories "3", "4" that are considered medium grades or non-investment groups and "7" that is the default. The categories of "5", "6" and "7" are aggregated and become one category. The remaining rankings except for category "14" are aggregated to one category. These two aggregated groups are considered low grades or substantial risk groups. For computational convenience this rating scheme is replaced by quantitative indicators, "A+", "A", ..., "F". As shown in Table 3, there are 7 states among, which state "1" or default, F, for the lowest rating category or the highest risk firms, which is a terminal rating of a firm and "7" represents the highest rating category or the lowest risk firms, "A+". Table 3 below, shows the rating scheme that summarizes the approach. The firms are assigned to a given ranking depending on their score being less or greater than a given threshold $c_i,\ i = 1, ..., 6$ at the beginning of each year. Accordingly, the rankings of a firm change over time.

The new adjusted transition matrix, after aggregating the three categories of "5", "6"

Table 3: Rating Scheme

R.C		TH
7	A+	≤c6
6	A	c5≤...<c6
5	B+	c4≤...<c5
4	B	c3≤...<c4
3	C	c2≤...<c3
2	D	c1≤...<c2
1	F	<c1

and "7" to one category "C" and after aggregating the categories "8", "9", "10", "11", "12", "13" to one category "D", is given in Table 4:

Table 4: Adjusted Transition Matrix for Year 2007

2007	Issuers	A+	A	B+	B	C	D	F
A+	91	53.54	13.13	13.13	10.10	9.09	1.01	0.00
A	322	2.80	27.33	24.53	11.80	20.81	12.42	0.31
B+	1132	0.88	2.39	28.62	11.93	39.13	16.97	0.09
B	3181	0.03	0.16	2.77	23.45	50.09	23.19	0.31
C	18287	0.03	0.22	0.91	1.96	52.82	43.40	0.79
D	38163	0.03	0.03	0.13	0.34	11.81	84.68	2.99

The aggregated adjusted transition matrix keeps the structure of the original migration matrix. However, due to the aggregation, we observe that the probabilities of downgrades from categories "B+", "B", "C" and "D" to categories "C" and "D" are higher than, or close to the stability rates of these ratings over the active sampling period. For instance, in Table 4 the frequency of transition from "B+" to "B+" is 28.62% but the frequency of downgrade from rating "B+" to rating "C" is 39.13% which is higher than stability rate of rating "B+". In the data set, we observe that pattern in the years 2007 and 2011 which proceeded the consecutive recessions in France when many firms were downgraded to lower categories. The two diagonals above and below the main diagonal show transition probabilities that are relatively high, while the other entries are close to zero. The frequencies of default are given in the last column of the migration matrix. As shown in Table 4, the probabilities of default increase monotonically with deteriorating credit quality. This is true for each of the 8 migration matrices in the data set. The original and aggregated adjusted transition matrices for the whole sampling period, 2007-2014, are reported in Appendix B.1 and B.2 respectively.

The number and the distribution of firms across the rating classes have changed over the sampling period from 2007 to 2014. Table 5 reveals that the total number of firms increased by 21% from $63,183$ to $76,589$ firms. The class "D" contains both category "D's" and "E's", which are relatively poor-quality firms. This change can be due to the fact that the average credit quality of the firms has decreased, or the rating system of the bank has become more strict.

Table 5: Distribution of Firms across Rating Classes in 2007 and in 2014

	A+	A	B+	B	C	D	Total
2007	91	322	1132	3181	18287	38163	63183
2014	10	105	410	1005	14670	58375	76589

In the next step, let us examine the dynamics of the transition matrices. We consider four-time series i.e. the time series of frequencies of 1) staying in the same class (called the stability rate, henceforth), 2) upgrade to a higher class, 3) downgrade to lower category and 4) the time series of the rate of default. We also display the average default rate of each rating category.

4.0.1 Stability Rate of Risk Categories

The main diagonal of each transition matrix indicates the frequency of staying in the same class of ratings. Figure 1 presents the time series of stability rates for all rating classes.

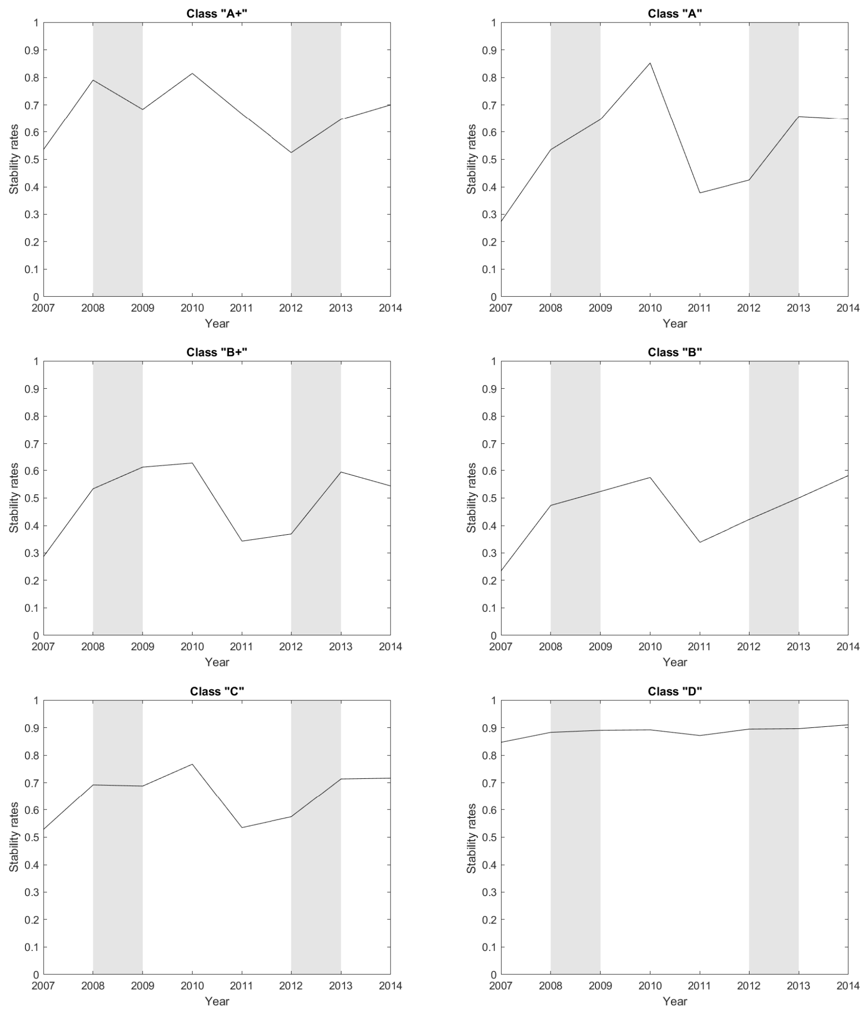

Figure 1: Stability Rates. This figure shows the time series of probabilities of staying in the same class. The migration rates are given in percentages on the vertical axes. The shaded areas indicate the OECD recessions in France.

As shown in Figure 1, the probability of staying in the rating class "A+" varies between 50-82%, while for the rating classes "A" and "B+", it varies between 28-86% and 29-63%

respectively. All rating classes show an upward trend in their stability rates before the economic recession in France in 2008-2009 and a downward trend in their stability rates before the economic recession in France and the Euro area in 2011-2013 as documented by the OECD.

4.0.2 Downgrade and Upgrade Probabilities

The numbers to the left and right of the main diagonal of a transition matrix represent the upgrading and downgrading rates respectively. For instance, the downgrade rates are defined as: $d_{l,t} = \sum_{k=1}^{l+1} \hat{p}_{l,k,t}$ and the upgrade rates are defined as: $u_{l,t} = \sum_{k=1}^{l-1} \hat{p}_{l,k,t}$. Figure 2 shows the time series of downgrades for each rating class. Figure 3 presents the upgrade rates for each rating class. We observe that the migration rates are changing over time.

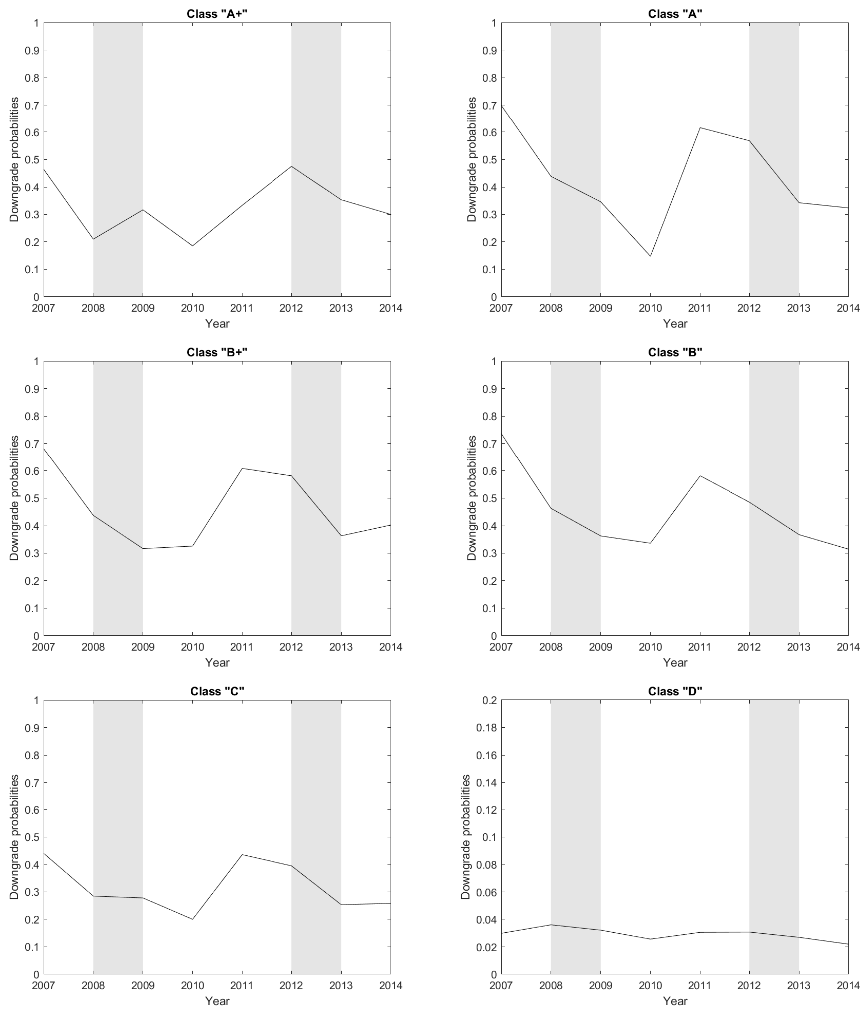

Figure 2: Downgrade Rates. This figure shows the time series of frequencies of downgrade for each rating class. The migration frequencies are given in percentages on the vertical axes. The shaded areas indicate the OECD recessions in France.

In Figure 2, the rating category "A+" reveals an overall increase in downgrade rates during the economic recession periods of 2008-2009 in France. All rating categories show an increase in downgrade rates before the economic recession in 2012-2013 in France. Figure 3 demonstrates the upgrade rates for each rating category. For instance, rating class "A+" shows downward trends in upgrade rates during the economic recessions 2008-2009 and 2012-

2013 in France.

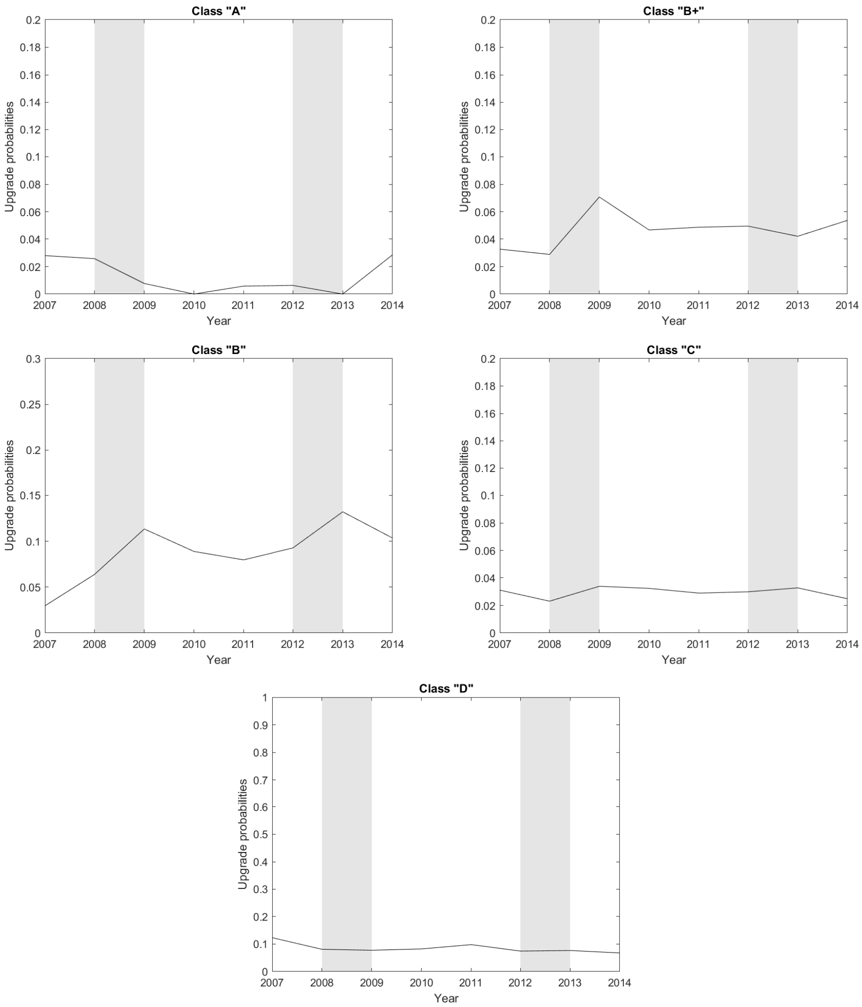

Figure 3: Upgrade Rates. This figure shows the time series of frequencies of upgrade for each rating class. The migration rates are given in percentages on the vertical axes. The shaded areas indicate the OECD recessions in France.

4.0.3 Probability of Default

Figure 4 displays the time series of rates of default from rating categories "B+", "B", "C" and "D". We plot the time series of default rates for rating class "B+", "B", "C" and "D" only, because the "A+" and "A" categories have null default rates. We observe that the rates of default for rating categories "B+", "B", "C" and "D" have an upward trend before the economic recession in 2008-2009 in France. Moreover, these rating categories have a downward trend before the other economic recessions in France in 2012-2013.

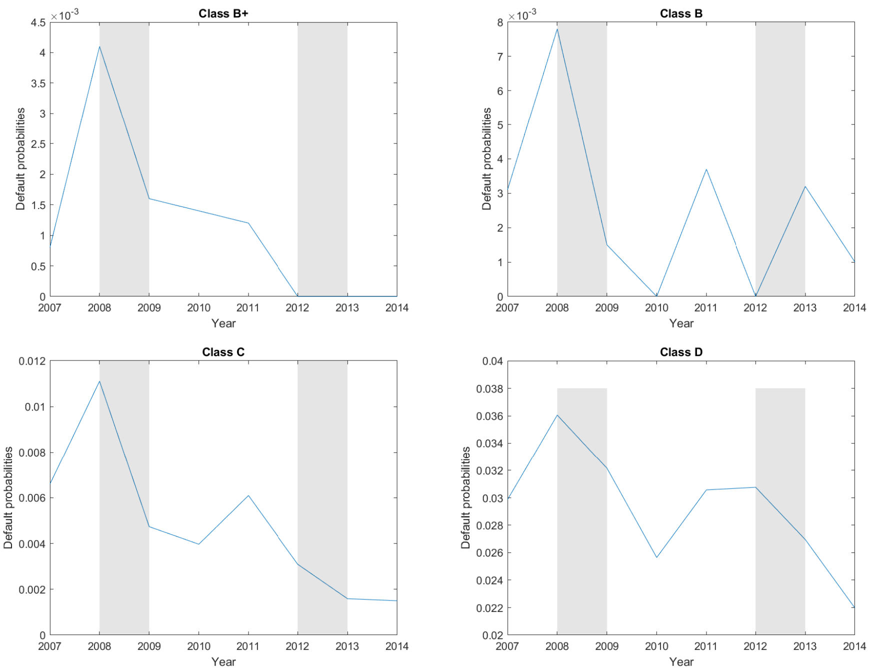

Figure 4: Probability of Default. This figure shows the time series of probabilities of default for rating class "B+", "B", "C" and "D". The default probabilities are given in percentages on the vertical axes. The shaded areas indicate the OECD recessions in France.

Figure5 displays the average default rates of each rating category. The horizontal axis shows the rating classes. Rating 7 refers to rating class "A+" and rating 2 corresponds to rating class "D". The average default rate increases as the rating class become riskier.

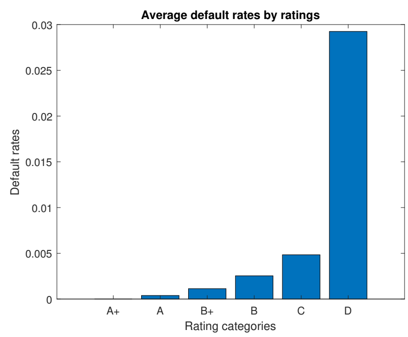

Figure 5: Average Default Rates by Ratings.This figure shows the average frequencies of default for each rating class. The default rates are given in percentages on the vertical axes. The shaded areas indicate the OECD recessions in France.

In the next section, the transition matrices for $t = 1, ..., T$ are used in the estimation of the stochastic factor ordered-Probit model with $k = 7$ rating classes.

5 Estimation Results

This section presents the estimation of the stochastic model described in Section 2, using the granularity-based maximum likelihood methods discussed in Section 3. Section (5.1), provides the estimation results on the linearly approximated state-space model that is used as a benchmark model to provide parameter values for the two-step estimation procedure. In Section (5.2), we provide the two-step estimation results.

5.1 Linear Approximation State-space Model-Benchmark Model

Let us write the ordered Probit stochastic transition model as an approximate linear state-space model and apply the procedure explained in Section 3.1. As explained earlier, before estimating the parameters we need to determine how many latent factors are driving the model. To do so, the principal component analysis (PCA), has to be performed. The PCA is the spectral decomposition of square $(T \times T)$ matrix of the series of canonical factors $\hat{a}_{l,k,t}$ and it is based on the analysis of eigenvalues and eigenvectors of this matrix. The PCA converts a set of correlated observed variables into a set of linearly uncorrelated variables by using orthogonal transformations. This transformation provides the eigenvectors corresponding to the eigenvalues of the variance-covariance matrix arranged in ascending order. In other words, the first principal component is the one associated with the largest eigenvalue. The motivation for doing this is to find the number of factors that have caused changes in migration probabilities across the rating classes. Therefore, the first objective is to compute the canonical factors $\hat{a}_{l,k,t}$ from the observed frequencies $\hat{p}_{l,k,t}$. The next step is to define the square matrix of series of these canonical factors which has the dimension $(T \times T)$ too. Then, using the principal component analysis (PCA) we can sort the eigenvalues in decreasing order and determine the associated eigenvectors. The ordered qualitative stochastic transition model with $K = 7$ is estimated as follows:

First, the model is re-parametrized to compute the canonical factors $\hat{a}_{l,k,t}$ from the observed frequencies $\hat{p}_{l,k,t}$. There are eight years $(T = 8)$ of transition probabilities matrices. Next, the cumulative probabilities of these frequencies are computed. Then the quantile function of a standard normal distribution is applied to obtain the canonical factors. In the next step, the matrix (YY'), which is the $(T \times T)$ matrix of series of estimated canonical factors is computed and the principal component analysis (PCA) is performed. These eigenvalues are reported in Tables 6:

Table 6: The Eigenvalues

10.4834	2.6814	1.1795	0.7452	0.4709	0.4003	0.1393	-5.6669e-16

These eigenvalues are derived from the spectral decomposition of the sample variance-covariance matrix of the series $\hat{a}_{l,k,t}$. The difference between the largest eigenvalue and the next one is significantly greater than the remaining differences. Moreover, we observe that the first eigenvalue is much larger than the other ones. Therefore, there is a cut-off in the sequence of eigenvalues. This evidence based on the PCA analysis suggests us to consider the stochastic model with one factor. It means that each year there is a common factor for all rating classes that drives the migration probabilities. The estimates of micro-parameters are displayed in Table 7.

Table 7: Micro-parameter Estimates-Linear Approximation

$\hat{c}_1 = -2.434$	$\hat{c}_2 = -1.333$	$\hat{c}_3 = -0.111$	$\hat{c}_4 = 1.384$	$\hat{c}_5 = 2.345$	$\hat{c}_6 = 2.669$

$\hat{\delta}$		$\hat{\beta}$		$\hat{\sigma}$	
1	$\hat{\delta}_2 = -2.553$	1	$\hat{\beta}_2 = 0.054$	1	$\hat{\sigma}_2 = 0.880$
2	$\hat{\delta}_3 = -1.085$	2	$\hat{\beta}_3 = 0.123$	2	$\hat{\sigma}_3 = 0.510$
3	$\hat{\delta}_4 = 0$	3	$\hat{\beta}_4 = 0.353$	3	$\hat{\sigma}_4 = 1$
4	$\hat{\delta}_5 = 0.715$	4	$\hat{\beta}_5 = 0.451$	4	$\hat{\sigma}_5 = 1.336$
5	$\hat{\delta}_6 = 3.916$	5	$\hat{\beta}_6 = 1.931$	5	$\hat{\sigma}_6 = 4.111$
6	$\hat{\delta}_7 = 5.797$	6	$\hat{\beta}_7 = 1.894$	6	$\hat{\sigma}_7 = 6.838$

Estimated Micro-parameters for the Factor Ordered-Probit Model. Thresholds $\hat{c}$, intercepts $\hat{\delta}$, factor sensitivities $\hat{\beta}$, and volatilities $\hat{\sigma}$ are displayed.

The thresholds are increasing functions of the rating class indexes. The estimated intercepts, factor sensitivities, and volatilities, reported in the middle panel for rows $l = 2, ..., 7$ of the transition matrix and correspond to rating class "A+", "A", ..., "C". As the rating category increases, the intercepts increase too. It confirms that the rating class "A+", which is the least risky class has the score function larger than other rating classes. The estimated factor sensitivities are all positive which means that when the factor increases the underlying scores for each rating class are improved. Lastly, the least risky rating "A+" has the highest volatility.

Next, we discuss the results of the two-step estimation method for the stochastic factor ordered-Probit model introduced in subsection 3.2 with $k = 7$ rating classes. This method relies on a simplified log-likelihood. We use the estimates of the linear approximation as a benchmark model that provides initial values in the numerical algorithm to maximize the log-likelihoods in the two-step approach.

5.2 Two-Step Efficient Estimators Approach

In the first step, we maximize the latent micro-likelihood functions L^{CS} given by equation (3.9) with respect to the factor values given that $\hat{\theta}_B$ (the micro-parameters obtained by the linear approximation state-space model) is known as follows:

- Step 1:

$$\hat{f}_{n,t}(\theta) = \arg\max_{f_t} \sum_{i=1}^{n} log f(y_{i,t}|y_{i,t-1}, f_t; \hat{\theta}_B).$$

In the second step, these solutions are reintroduced in the latent micro-likelihood functions L^{CS} and aggregated over time to obtain the estimator of θ as follows:

- Step 2:

$$\hat{\theta}^*_{n,T} = \arg\max_{\theta} \sum_{t=1}^{T} \sum_{i=1}^{n} log f(y_{i,t}|y_{i,t-1}, \hat{f}_{n,t}(\theta); \theta).$$

Next, we reintroduce the estimates of $\hat{\theta}^*_{n,T}$ into the latent micro-likelihood functions to obtain the factor values which are used as proxies for the unobserved factor values as follows:

$$\hat{f}_{n,T,t} = \hat{f}_{n,t}(\hat{\theta}^*_{n,T}).$$

The estimator of the macro-parameter ρ can be derived simply by applying the Maximum Likelihood (ML) estimation on the autoregressive model as follows:

$$\hat{f}_{n,T,t} = \mu + \rho \hat{f}_{n,T,t-1} + \eta_t, \quad \eta_t \sim IIN(0, \sigma_\eta^2), \quad t = 1, ..., T.$$

This autoregressive model can also be estimated by the ordinary least square (OLS) estimator which coincides with the ML estimator given the normality. We impose the identification restrictions for rating class "B" as follows:

$$c_3 = \delta_4 = 0, \text{ and } \beta_4 = \sigma_4 = 1,$$

Table 8 displays the estimates of the micro-parameters which are all statistically significant, except for the factor sensitivity β_2 of rating category "D". The upper panel shows the estimates of the thresholds parameters c_k. As expected, they increase with the rating class indices. Moreover, we can see that the thresholds decrease with rating quality. The estimates for the parameters in rows $l = 2, ..., 7$ of the transition matrix are displayed in the lower panel.

Table 8: Micro-parameter Estimates-Two-step Efficient Estimation Method

$\hat{c}_1 = -5.141^{***}$ (0.026)	$\hat{c}_2 = -1.840^{***}$ (0.011)	$\hat{c}_3 = 0$	$\hat{c}_4 = 1.032^{***}$ (0.006)	$\hat{c}_5 = 2.143^{***}$ (0.014)	$\hat{c}_6 = 3.350^{***}$ (0.023)

$\hat{\delta}$		$\hat{\beta}$		$\hat{\sigma}$	
1	$\hat{\delta}_2 = -3.213^{***}$ (0.055)	1	$\hat{\beta}_2 = -0.011$ (0.022)	1	$\hat{\sigma}_2 = 1.033^{***}$ (0.011)
2	$\hat{\delta}_3 = -1.752^{***}$ (0.010)	2	$\hat{\beta}_3 = 0.223^{***}$ (0.001)	2	$\hat{\sigma}_3 = 0.924^{***}$ (0.005)
3	$\hat{\delta}_4 = 0$	3	$\hat{\beta}_4 = 1$	3	$\hat{\sigma}_4 = 1$
4	$\hat{\delta}_5 = 0.633^{***}$ (0.009)	4	$\hat{\beta}_5 = 0.089^{***}$ (0.002)	4	$\hat{\sigma}_5 = 1.380^{***}$ (0.009)
5	$\hat{\delta}_6 = 1.570^{***}$ (0.016)	5	$\hat{\beta}_6 = 0.189^{***}$ (0.002)	5	$\hat{\sigma}_6 = 1.564^{***}$ (0.011)
6	$\hat{\delta}_7 = 10.925^{***}$ (0.055)	6	$\hat{\beta}_7 = 1.124$ (0.005)	6	$\hat{\sigma}_7 = 10.230^{***}$ (0.059)

Estimated Parameters of the Factor Ordered-Probit Model. Thresholds $\hat{c}$, intercepts $\hat{\delta}$, factor sensitivities $\hat{\beta}$, and volatilities $\hat{\sigma}$. Standard errors are given in parentheses. (* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$).

The intercepts δ_l are increasing with respect to the rating index, and increasing with respect to rating quality which confirms that the underlying quantitative score $y_{i,t}^*$ for credit quality is larger for the less risky rating classes. It also shows that the downgrade risk is higher for lower credit ratings. The sensitivity β_2 of the rating category "D" to the common factor is not statistically significant. Thus, the sensitivities β_l of all rating classes to the common factor are positive. This shows that an increase in the factor increases the underlying quantitative score for credit quality in all rating classes. Generally, the riskier rating categories have smaller idiosyncratic volatility, σ_l.

The approximated factor values are given in Table 9. In order to obtain mean zero and unit variance, these values have been standardized.

Table 9: Approximated Factor Values

Estimated factor values							
2007	2008	2009	2010	2011	2012	2013	2014
-1.8238	-0.0424	0.5360	1.0450	-0.9329	-0.3803	0.7928	0.8057

Table 10 shows the estimated macro-parameter $\hat{\rho}$ which is obtained by the Maximum Likelihood estimation of the autoregressive equation (3.12). The estimator is based on the cross-sectional approximations of the factor values $\hat{f}_{n,T,t} = \hat{f}_{n,t}(\hat{\theta}^*_{n,T})$.

Table 10: Estimated Macro-coefficient

	Estimate	SE	T.Stat	P-Value
μ	0.2180	0.2918	0.7471	0.4549
ρ	0.4097	0.3027	1.3534	0.1759
σ^2_η	0.2740	0.3880	0.7060	0.4801

The estimator of parameter σ^2_η is given by $\hat{\sigma}^2_\eta = \frac{1}{T-1}\sum_{t=2}^{T}\hat{\eta}^2_t$, where $\hat{\eta}_t = \hat{f}_{n,T,t} - \hat{\mu} - \hat{\rho}\hat{f}_{n,T,t-1}$ are the residuals. The autoregressive coefficient is positive but not statistically significant. The approximated factor values $\hat{f}_{n,T,t}$ are displayed in Figure 6.

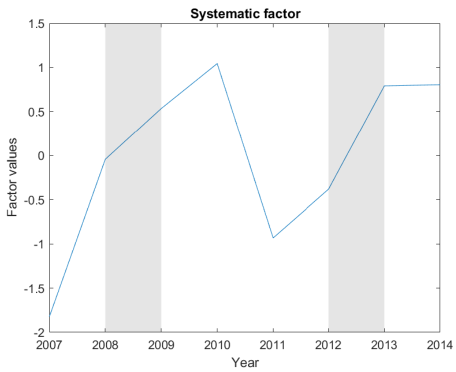

Figure 6: Systematic Factor. The figure displays the pattern of the systematic factor $\hat{f}_{n,T,t}$ for $t = 1, ..., 8$. The factor estimates are standardized to obtain zero-mean and unit variance in the sample. The shaded areas indicate the OECD recessions in France.

The cyclical pattern of the systematic factor in Figure 6 is consistent with the evolution of migration probabilities shown in Figures 1, 2, 3 and 4. As we stated before, the troughs in the factor pattern are associated with the troughs of stability rates, the peaks in downgrade rates in Figures 1 and 2. Moreover, the factor path features a downward local trend in 2010-2011 corresponding to decreases in the downgrade risk and an upward local trend in the years 2007-2010 corresponding to an increase in the downgrade risk.

The next section discusses the link between the approximated factor values with the business cycle literature. The existing literature usually compares the common factor with the underlying state of the economy to interpret the factor. It is very common to link the approximated factor path with the macro variables such as the business cycle, inflation rate, industrial production, unemployment or interest rate. There exists several studies which relate the failure rate with proxies of the U.S. business cycle [see, Nickell et al. (2000), Bangia et al. (2002) and Rösch (2005)].

5.2.1 Macroeconomic covariates

n order to find a link between the systematic factor f_t and the economic variables, we group the macroeconomic variables in four categories of, national accounts, financial markets, stock markets, and composite indices. We include variables that have been explored in previous research [see Kim (1999), Figlewski et al. (2012)]. Corporate credit risk is influenced by a variety of broad economic conditions. Thus, we select several key economic indicators to test and find the link between our systematic factor and macroeconomic variables.

National Accounts

Real GDP growth: The time series of real GDP is obtained from the St. Louis Fed. The change in GDP refers to economic strength. Higher GDP growth might influence the firm's income and lower GDP growth increases the possibility of default for firms.

Real GDP actual minus potential: The time series of the real GDP actual minus potential for France is estimated by the St. Louis Federal Reserve. Although the average of the series is negative (algebraically) but, stronger economic conditions cause a higher value for the real GDP actual minus potential.

Industrial production: We include the Production of Total Industry as a better measure for the corporate sector since the GDP comprises all economic activity, including government, non-corporate business, and other sectors.

Unemployment and employment rate: The most visible measures of the overall health of the economy are unemployment and employment rates. low unemployment should decrease the probability of downgrade migrations and default rates. A high employment rate also should increase the probability of upgrade migrations.

Inflation rate: There exist a variety of price indices that measure inflation in an economy. We include the yearly percentage change in the seasonally adjusted Consumer Price Index (CPI) and Producer Price index (PPI). There is a common perception that inflation is bad for the economy, thus, it is possible to have more default rates for firms. However, the real value of the outstanding debt of the firm in terms of nominal dollars could reduce as a result of inflation, therefore, it is possible to have lower rates of default for firms. [Figlewski et al. (2012)]

Financial Markets

Interest rate: We include short-term and long-term Interest rates. The long-term interest rates are considered as a measure of the overall level of the interest rate at longer maturity in an economy and the short-term interest rate is considered as a measure of the tightness of the money market. Duffie et al. (2007) found a negative relation between the firm's credit default and a short-term interest rate. The negative relation can be explained by the fact that central banks increase the short rates to cool down business expansion. We will have more optimistic markets in an economy when we have higher long-term interest rates.

Spread: We include the spread between short-term and long-term interest rates.

Exchange rate: Firms with a high volume of international business are expected to be more affected by exchange rates. If the exchange rate is high, the importing businesses are expected to be positively affected and exporting businesses are negatively affected.

Stock Markets

CAC 40 return: The general health of the corporate sector can be indicated by the performance of the stock market. Moreover, the level and volatility of the firm's stock price have a direct effect on a firm's default risk exposure in a structural model. We include the return of the CAC 40 index. The probability of default and downgrade is negatively related to the stock market return.

CAC 40 volatility: We include the volatility index of CAC 40 as a measure of market risk. The probability of default is positively related to volatility.

Composite Index

Composite leading indicator (CLI): There exist previous studies that showed a strong link between some comprehensive index and corporate default risk. Therefore, we choose the composite leading indicator (CLI) as a short-term measure of economic movement in a qualitative term. The early signals of turning points in business cycles showing the fluctuation of the economic activity around its long term potential level cab be provided by the CLI.

These variables are collected from various sources including the St. Louis FRED Economic Data, the World Bank database and the OECD database for France over the period of 2007-2014. These variables are shown in Table 11. The descriptive statistics of these macroeconomic variables are given in Appendix C.

Table 11: Description and Sources of Macroeconomic Variables

Category	**Variables**	**Description**	**Resources**
National accounts	GDP	Real GDP Growth Rate (annual %)	World Bank
	OUTG	Output Gap, % of Potential GDP (Actual GDP minus Potential)	St. Louise FRED Economic Data
	UNEM	Unemployment Rate (% of total labor force)	OECD database
	EMP	Employment Rate (% of total labor force)	OECD database
	IPROD	Production of Total Industry	OECD database
	CPI	Consumer Price Index	St. Louise FRED Economic Data
	PPI	Producer Price Index	St. Louise FRED Economic Data
Financial markets	SINTS	Short-term Interest Rate	OECD database
	LINTS	Long-term Interest Rate	OECD database
	SPL-S	Spread between Short and Long-term Interest Rate	OECD database
	EXCH	Exchange Rate	OECD database
Stock markets	CAC40	Return of CAC 40 Index	Yahoo finance
	VCAC4	Volatility of CAC 40 Index	yahoo finance
Composite index	CLI	Composite Leading Index	OECD database

The next step is to calculate the correlation between the systematic factor and these variables. However, the time dimension T of our data set is small, therefore, the estimated correlation between the systematic factor and these variables may not be accurate. Moreover, due to the non-linear pattern of the systematic factor and these variables the sample correlation may not be sufficient. Thus, we do not rely just on the correlation and consider the graphical analysis as well.

The correlation between the variables in Table 11 and the systematic factor are reported in Table 12. Next, we compare the evolution of variables that have patterns similar to the systematic factor in Figure 6.

Table 12: Correlation between the Macroeconomic Variables and the Systematic Factor

	Macroeconomic variables	Correlation with the systematic factor	P-Value
National accounts	GDP	-0.41	0.31
	OUTG	**-0.76*****	**0.02**
	UNEM	0.53	0.17
	EMP	-0.3	0.46
	IPROD	**-0.73*****	**0.03**
	CPI	0.47	0.22
	PPI	0.12	0.77
Financial markets	**SINTS**	**-0.72*****	**0.04**
	LINTS	-0.57	0.13
	SPL-S	0.52	0.18
	EXCH	-0.27	0.5
Stock markets	CAC40	0.22	0.59
	VCAC40	0.33	0.41
Composite index	CLI	-0.51	0.18

In terms of the correlation between the systematic factor and the variables, we observe that the output gap and the total industrial production from the national accounts group have a high negative correlation significant at 5% with the systematic factor. From the financial markets group, the systematic factor is inversely correlated with the short-term interest rate and this correlation is significant at 5%. These three variables have a statistically significant correlation with the systematic factor.

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Let us first take a look at the evolution of the output gap, total industrial production, from the national accounts group, the short-term interest rate from the financial markets and the annual return on CAC 40 from the stock markets group in France over the period 2007-2014. When we plot the variables that have a negative correlation with the systematic factor, we multiply the systematic factor by (-1) in order to have a better vision of the factor path and we keep the original factor in the case of CAC 40 which has a positive correlation with the factor.

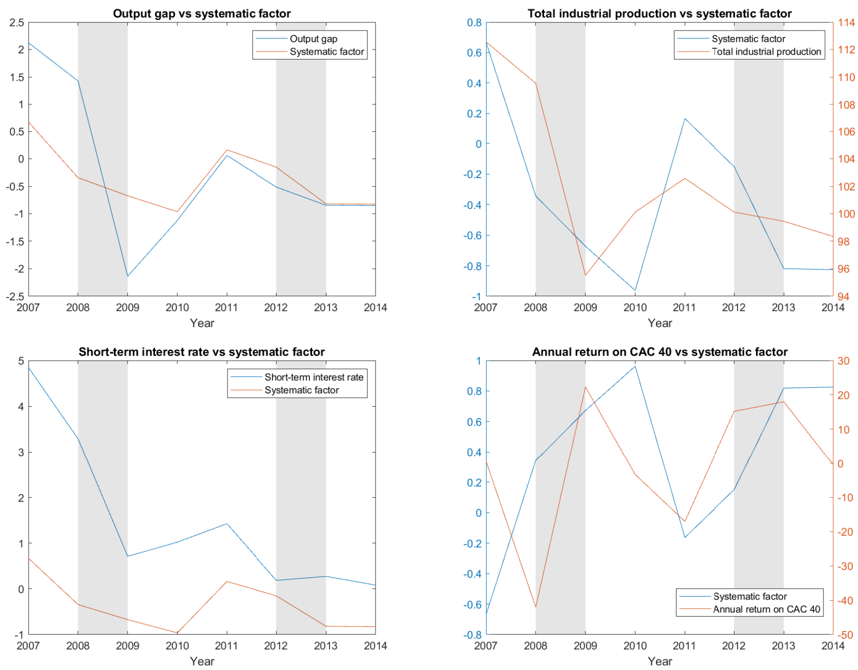

Figure 7: Correlation between Macro-variables and the Systematic Factor. This figure shows the evolution of the output gap % of Potential GDP, the total industrial production, the short-term interest rate, per cent, per annum (%), and the annual return on CAC 40 for France over the period 2007-2014 and the systematic factor. The shaded periods refer to OECD recessions in France.

It can be easily seen that the cyclical pattern of the negative factor is close to the output gap, the total industrial production and the short-term interest in France. Moreover, the systematic factor has a pattern similar to the annual return on the CAC 40. The output gap shows local downward trends during both recessions periods in France and since our factor is negatively correlated with the output gap we observe local upward trends in the systematic factor during the recession periods. In 2011, the output gap has a peak and the systematic factor has a trough in 2011 as well. We observe the same pattern for the total industrial production and the short-term interest rate as well. The CAC 40 index which is a benchmark French stock market shows local upward trends during both recessions which coincide with upward trends in the systematic factor. Both series have a trough in 2011 as well.

In order to see how these variables are related to the systematic factor, we perform the linear regressions. These four aforementioned variables are regressed on the systematic factor. The results of the linear regressions are reported in Table 13.

Table 13: Linear Regression- Two-step Estimation Method

Variables		Estimate	SE	T.Stat	P-Value	R-Squared
OUTG	β_0	0.45	0.41	1.09	0.31	0.58
	β_1	**-1.87***	0.64	-2.90	0.02	
IPROD	β_0	**105.03***	1.84	57.07	0.00	0.53
	β_1	**-7.48***	2.84	-2.63	0.03	
SINTS	β_0	**2.27***	0.55	4.14	0.01	0.52
	β_1	**-2.17***	0.85	-2.55	0.04	
CAC 40	β_0	-3.89	9.53	-0.40	0.69	0.05
	β_1	8.31	14.74	0.56	0.59	

The regression equation is as follows:

$$Y_t = \beta_0 + \beta_1 f_t + \epsilon_t, \ E_{\epsilon_t} = 0, \ Var(\epsilon_t) = \sigma^2$$

The slopes β_1 are statistically significant at 5% for the output gap, the total industrial production, and the short-term interest rate and the intercepts β_0 are statistically significant at 5% for the total industrial production and the short-term interest rate.

This regression analysis and the comparison of patterns of the variables displayed in Figure 7 suggest that the systematic factor obtained from the two-step efficient estimation approach is related to these variables. In addition to this analysis, we will perform the stress testing in the next Section which allows us to interpret the systematic factor in more detail.

6 Fitted Credit Migration Matrices

To assess the fit of the model, we compare the observed and the estimated transition matrices. The Euclidean distance or Euclidean metric is one way to compare two matrices. This mathematical method is based on the straight-line distance between two points in the Euclidean space. This method computes the average absolute difference between the corresponding elements of two matrices. Moreover, one can measure the average root-mean-square difference between multiple corresponding elements of the matrices [see, Israel et al. (2001), Bangia et al. (2002)]. However, the Euclidean metric provides a relative measure of the distance between two matrices and it is difficult to judge whether this distance is large or small. Moreover, it is not easy to interpret the economic meaning of the distance. Another method that allows us to compare two transition matrices is to use a mobility index [see, Geweke et al. (1986)]. Jafry and Schuermann (2004) extends that method and introduces a singular value metric (SVD-mobility index) which is a function that compares two migration matrices with respect to their ability to generate migration events.

In order to assess and compare, we proceed in two steps. First, we compute the estimated transition probabilities from the two-step efficient estimation method and compare the evolution of fitted stability rates, downgrade and upgrade rates of each rating category during the sample period with the evolution of observed stability rates, downgrade and upgrade rates for each rating class. Second, we compare the transition matrices implied by the two-step efficient estimation method with the observed migration matrices using the SVD-mobility index introduced by Jafry and Schuermann (2004). The estimated transition matrices obtained by the two-step efficient estimation method for the whole sample period are reported in Appendix D.

6.1 Comparison using the Stability, Downgrade and Upgrade Rates

First, we compute the implied probabilities of staying in the same rating class known as the implied stability rates. The evolution of the implied stability rates and the empirical stability rates for each rating class is presented in Figure 8.

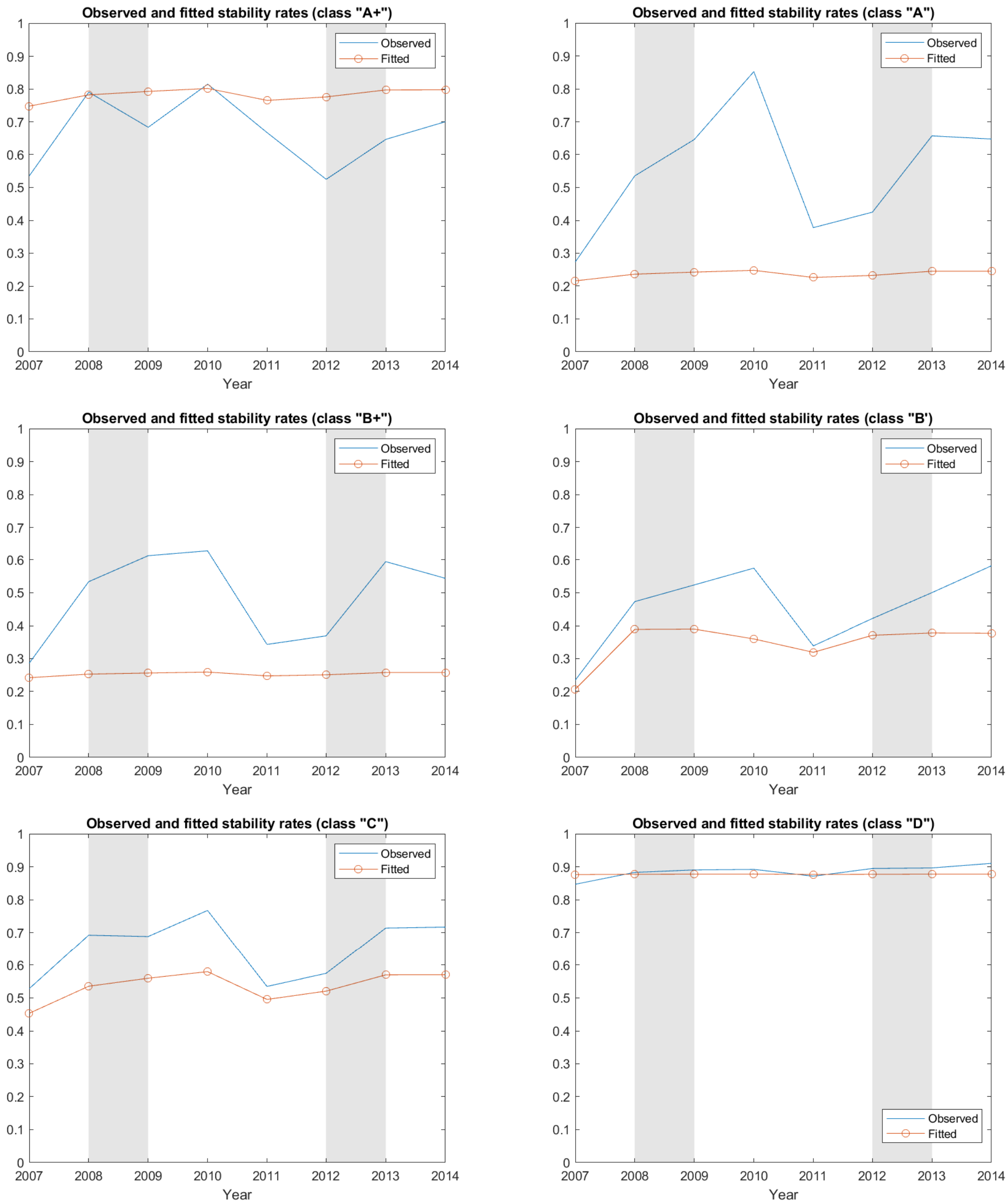

Figure 8: Observed and Fitted Stability Rates. This figure shows the time series of observed probabilities of staying in the same class (solid line) and fitted probabilities of staying in the same class (circles). The migration probabilities are given in percentages on the vertical axes. The shaded areas indicate the OECD recessions in France.

The pattern of the fitted stability rates captures the pattern of observed stability rates especially in the rating classes "B", "C", and "D". Figure 9 displays the observed and fitted downgrade probabilities for each rating class. Figure 10 demonstrates the observed and fitted upgrade probabilities for each rating class.

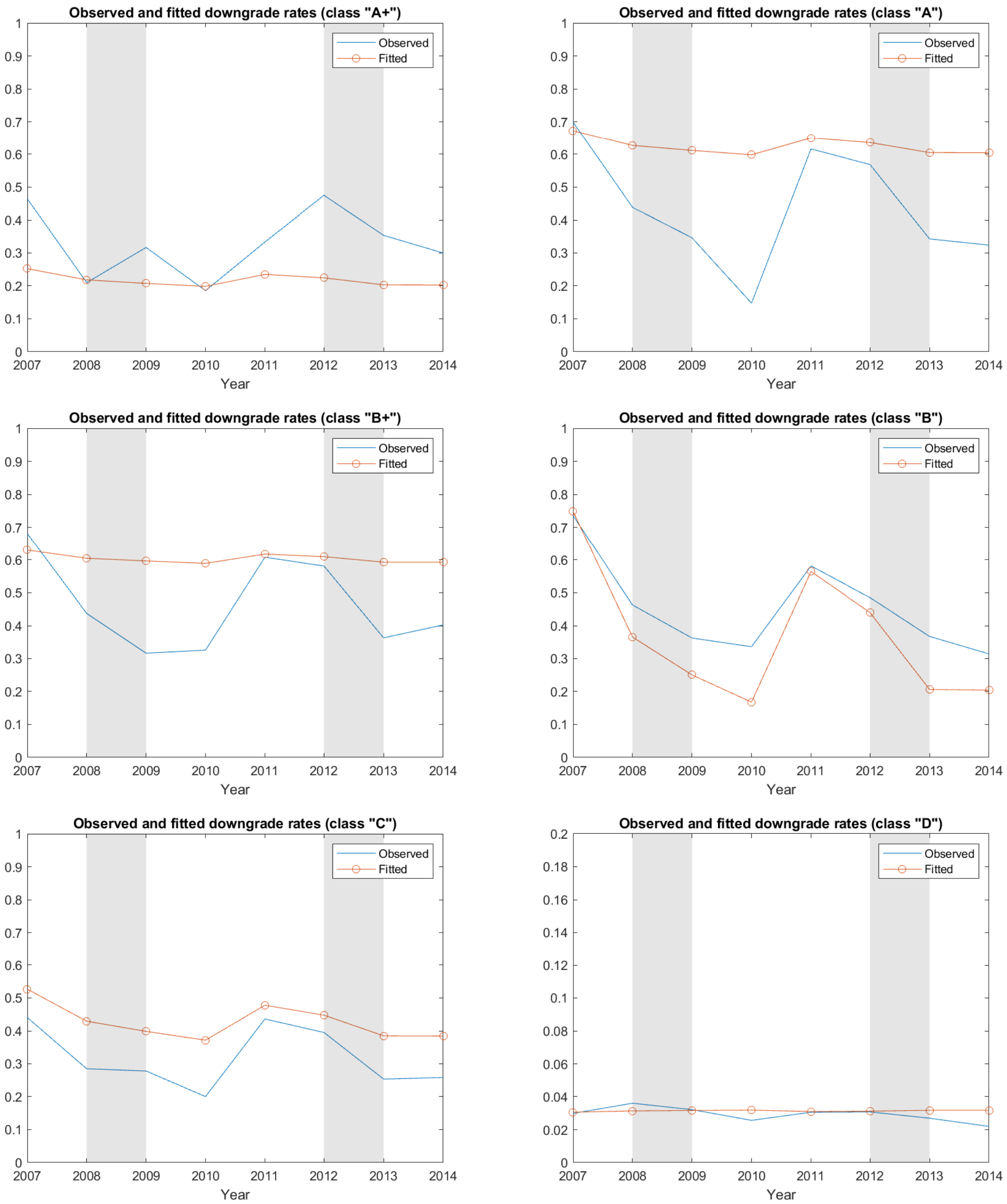

Figure 9: Observed and Fitted Downgrade Rates. This figure shows the time series of observed probabilities of downgrade for each rating class (solid line) and fitted probabilities of downgrade for each rating class (circle). The migration probabilities are given in percentages on the vertical axes. The shaded areas indicate the OECD recessions in France.

The pattern of the fitted downgrade probabilities in all rating classes captures the pattern of observed downgrade probabilities which confirms that our model is successful in capturing the downgrade risk.

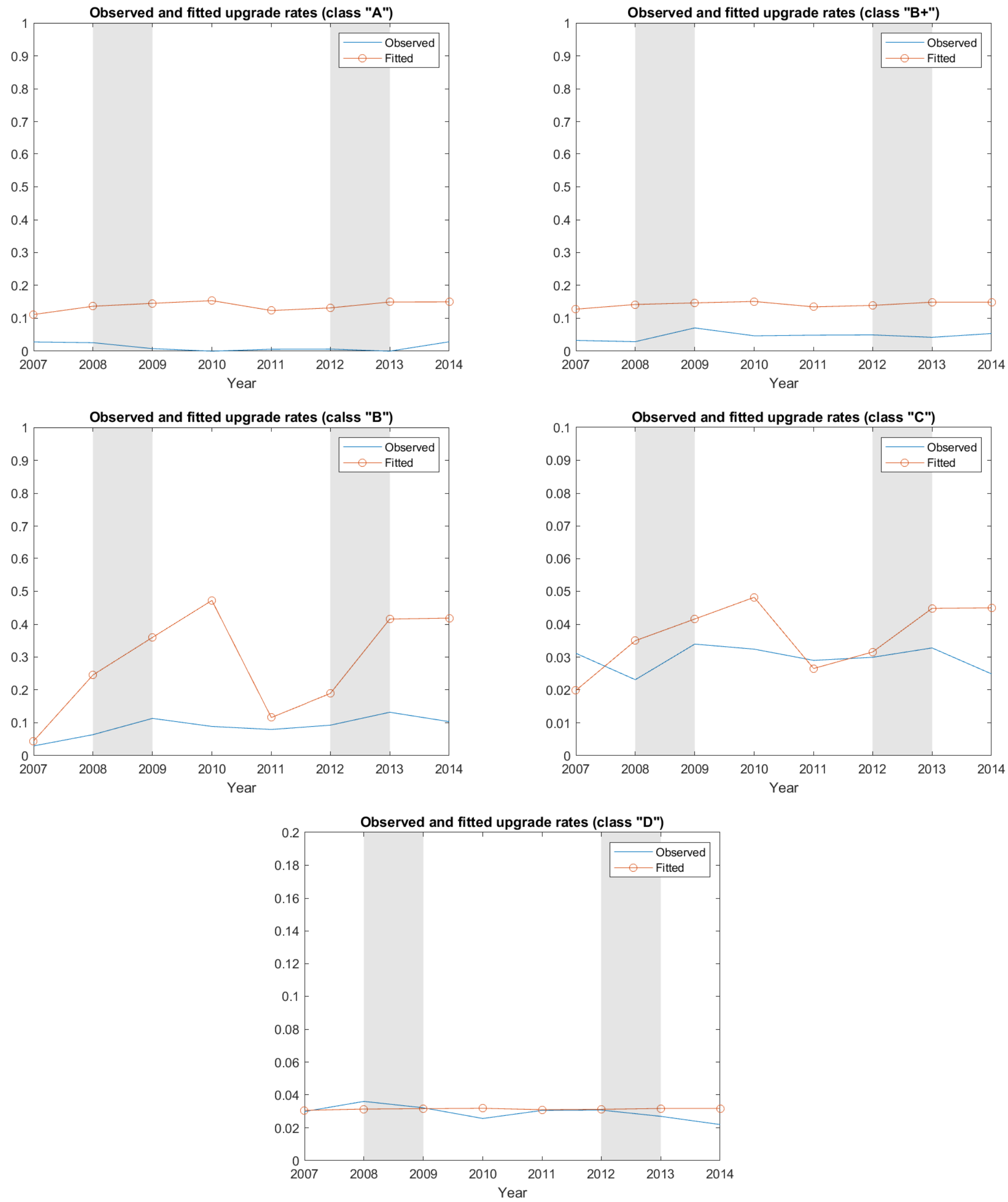

Figure 10: Observed and Fitted Upgrade Rates. This figure shows the time series of observed probabilities of upgrade for each rating class (solid line) and the fitted probabilities of upgrade for each rating class (circle). The migration probabilities are given in percentages on the vertical axes. The shaded areas indicate the OECD recessions in France.

Moreover, some actual and fitted migration probabilities from rating class "B", $l = 4$, to rating categories "A", "B+", "C", and "D", $k = 6, 5, 3, 2$ are displayed in Figure 11.

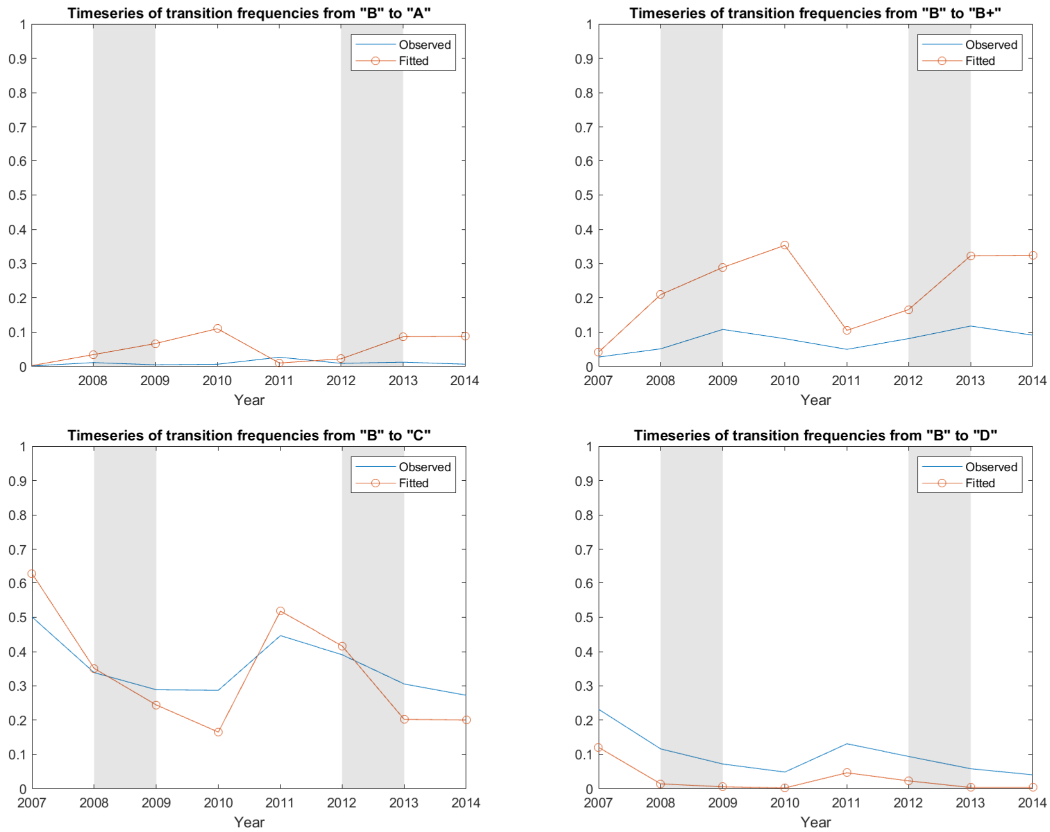

Figure 11: Time-series of Observed and Fitted Migration Probabilities from Rating Class 'B" to Rating Categories "A", "B+", "C", and "D", $k = 6, 5, 3, 2$. The migration probabilities are given in percentages on the vertical axes. The shaded areas indicate the OECD recessions in France.

The peaks and troughs in the observed and fitted time series of migration probabilities for rating class "B" to the ratings "A", "B+", "C", and "D" coincide in all panels of Figure 11.

Figures 8-11 indicate that the fitted probabilities capture the trends in the observed migration probabilities although, their values may differ. Next, we calculate the average vertical distance between the observed and fitted migration probabilities as follows:

$$Dis = \frac{1}{T}\sum_{t=1}^{T}(p_{l,k,t} - \hat{p}_{l,k,t})^2,$$

Table 14 shows the average distance between the observed and fitted stability, downgrade and upgrade rates.

Table 14: Average Distance between Observed and Fitted Migration Probabilities

	A+	A	B+	B	C	D
Stability rates	0.02	0.12	0.07	0.01	0.01	0.0003
Downgrade rates	0.02	0.05	0.03	0.01	0.01	2.26E-05
Upgrade rates	0	0.01	0.009	0.05	0.0001	0.0002

The best-fitted downgrade and the upgrade rates are for rating classes "D" and "C", respectively. The one-factor model is successful in reproducing the general pattern of the expected risk in each rating class. The relatively higher discrepancies for some rating classes may be due to the fact that some of the assumptions of the model may not hold. For example, the assumption of a homogeneous population of firms is a strong assumption. Another reason could be the choice of the cumulative distribution function Φ, which can be different from the Gaussian one, and feature different tail or skewness behaviors. Moreover, the time dimension T of the panel is short, which affects the accuracy of the estimators. Let us take a look at the estimated migration probabilities in the joint optimization approach. In the next section, we compare the fitted and observed migration probabilities obtained from the joint optimization.

6.2 Comparison using the SVD-Mobility Index

Jafry and Schuermann (2004) introduce a metric called the SVD-mobility index to compare two transition matrices on their ability to produces migration events. The SVD-mobility index is the most common method of comparison for migration matrices in the literature. The mobility index provides a scalar value that captures the overall dynamic features of given transition matrices. This is the advantage of the mobility index as instead of comparing the migration probabilities each year, we can compare the overall dynamics of the matrices. We use Jafry and Schuermann (2004) SVD-mobility index to compare the migration matrices implied by the two-step estimation and the joint optimization methods with the observed migration matrices in the data.

The SVD-mobility index $M_{SVD}(\mathbb{P})$ is defined as a function of transition matrix $\mathbb{P}$. The main feature in a transition matrix $\mathbb{P}$ is the amount of migration or "mobility" imposed on the state vector from one period to the next. We subtract the identity matrix $\mathbb{I}$ from the original matrix $\mathbb{P}$, to get the mobility matrix $\tilde{\mathbb{P}}$. The mobility matrix includes only the dynamic part of the original matrix $\mathbb{P}$ and reflects the "magnitude" of $\mathbb{P}$ in terms of the implied mobility since the main diagonal is the negative values of the sum of the row elements. Jafry and Schuermann (2004) find that the average of all the singular values of the mobility matrix $\mathbb{P}$ captures the general characteristics of that matrix. The SVD-mobility index for a d-dimensional matrix $\mathbb{P}$ is given by:

$$M_{SVD}(\mathbb{P}) = \frac{\sum_{i=1}^{d} \sqrt{\lambda_i(\tilde{\mathbb{P}}'\tilde{\mathbb{P}})}}{d}$$

where $\tilde{\mathbb{P}} = \mathbb{P} - \mathbb{I}$, and $\mathbb{P}'$ is its transpose. $\lambda_i(\tilde{\mathbb{P}}'\tilde{\mathbb{P}})$ is the i^{th} eigenvalue of $\tilde{\mathbb{P}}'\tilde{\mathbb{P}}$, sorted in decreasing order, i.e. $\lambda_1(\tilde{\mathbb{P}}'\tilde{\mathbb{P}}) > .... > \lambda_d(\tilde{\mathbb{P}}'\tilde{\mathbb{P}})$.
Suppose that the values of all diagonal elements of matrix $\mathbb{P} = \mathbb{P} - \mathbb{I}$ are $(1 - p)$ and all off-diagonal elements are equal to $(p/d - 1)$, where p represents the probability that a given state will undergo a migration (to any of the others). Then, Jafry and Schuermann (2004) show that the M_{SVD} metric applied to $\mathbb{P}$ yields the following exact result:

$$M_{SVD}(\mathbb{P}) = p,$$

This result indicates that the average migration probability p is numerically identical to the average singular value metric. For instance, if we get the value of the singular value metric for a given matrix, 0.1, say then we can conclude that the matrix has an effective average probability of migration of 0.1. Thus, the difference between the two migration matrices could be measured by M_{SVD} as follows:

$$D_{SVD}(\mathbb{P}_1, \mathbb{P}_2) = M_{SVD}(\mathbb{P}_1) - M_{SVD}(\mathbb{P}_2),$$

A directional deviation between two matrices in terms of the mobility or approximate average probability of migration can be measured by this difference.

To compare the mobility "size" of the migration matrices, we use the M_{SVD} index for the period of 2007-2015 for the Two-step estimation method. We applied the SVD mobility index to each matrix calculated according to the two-steps estimation. The results are shown in Table 15.

Table 15: M_{SVD}-Mobility Index by the Two-step Estimation Approach

Year	2007	2008	2009	2010	2011	2012	2013	2014
Two-steps	0.2577	0.2257	0.2214	0.2212	0.2400	0.2302	0.2209	0.2209
Observed data	0.2465	0.1581	0.1435	0.1145	0.2163	0.2063	0.1478	0.1437
D_{SVD}	0.0111	0.0676	0.0779	0.1067	0.0236	0.0239	0.0731	0.0772

Figure 12 shows the annually SVD-mobility index of the two-steps efficient estimation approach and the observed migration matrices.

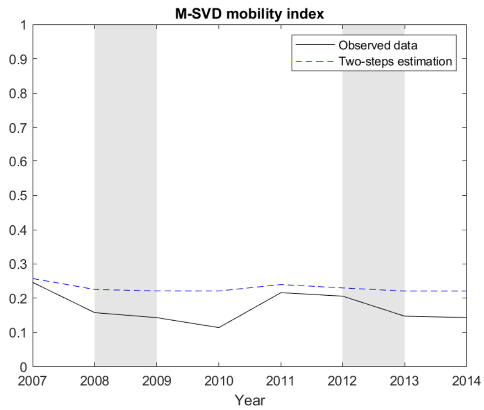

Figure 12: The M_{SVD} Mobility Index. This Figure exhibits the annual mobility of the migration matrices using the M_{SVD} for the period of 2007-2014. The two-step estimation approach: dashed line, the observed data: solid line. The shaded areas indicate the OECD recessions in France.

Overall, we can conclude that the two-step method capture well the frequencies of transition matrices as the M_{SVD} measures are close to the empirical M_{SVD} values.

In the next step, we perform the stress testing analysis in the factor stochastic migration model.

7 Stress Testing with the Stochastic Migration Model

As stated earlier in the text, estimations of probabilities of default (PD) and migration rates under hypothetical or historical stress scenarios are required by Basel II. Stress testing is an important tool to assess the financial stability of banks and financial institutions. Conducting regular regulatory stress testing exercises under the supervision of the European Banking Authority (EBA) is mandatory for financial institutions in Europe[3]. The aim is to determine the behavior of banks portfolios under stress conditions. In particular, the impact of extreme events or changes in market conditions on the capital adequacy of banks based on the scenarios of macroeconomic and financial shocks can be assessed by the stress testing analysis. Generally, credit risk stress tests concern risk parameters such as the probability of default at a one-year risk horizon (PD), migration probability, loss given default (LGD) or exposure loss at Default (EAD). These risk measures form the main building blocks of the Basel II regulatory capital supervision for credit risk. In our particular framework, the

[3]OSFI is the supervisory authority (Office of the Supervisor of Financial Institution)

stress test exercise focuses on the probability of migration from one credit rating grade to another, and especially, the probability of migrating to default.

We study the changes in the migration rates in our portfolio if the economy experiences a moderate downturn or upturn. For this purpose, a one-time shock with the size of one standard deviation of the factor (0.56) is added to the estimated systematic risk factor which affects the migration rates. The following hypothetical scenarios are considered:

- Scenario 1: A positive one-time shock to the systematic factor f_t in which the factor increases by one standard deviation of factor σ_f,
- Scenario 2: A negative one-time shock to the systematic factor f_t in which the factor decreases by one standard deviation of factor σ_f,

These scenarios are designed to resemble an improvement and deterioration of the economic situation. Furthermore, we consider two economic principles, "mutatis mutandis", ie "allowing other things to change accordingly" and "ceteris paribus", ie "all other things being equal" or "holding other factors constant" in our stress testing analysis. In the first case, i.e. "mutatis mutandis", the one-time positive and negative shocks are added to the systematic factor in the years 2008 and 2010 separately. These shocks have an effect on the systematic factor in those years as well as the factor values over the following years. In other words, we assume that the factor in years following the shocks returns to the estimated trajectory. In the other case, i.e. "ceteris paribus", we impose the two shocks to the systematic factor in the years 2008 and 2010, then, assume that over the next years after the shocks, the factor returns to a flat baseline trajectory. We examine the effects of shocks to determine how quickly each shock effects dissipate.

We consider the two above scenarios and consider the positive and negative shocks to the systematic factor in the years 2008 and 2010 separately. The stressed migrations under both assumptions and shock scenarios computed from the two-step estimate for the period 2010 to 2014 are reported in Appendix E.

7.1 Shocks to the Systematic Factor- Assuming "Mutatis Mutandis"

We add a positive and a negative one-time shock equal to one standard deviation of factor, σ_f, to the systematic factor in the years 2008 and 2010. Figures 13 and 14 show the paths of the systematic factor and of the stressed factor under both stress scenarios in the years 2008 and 2010 respectively.

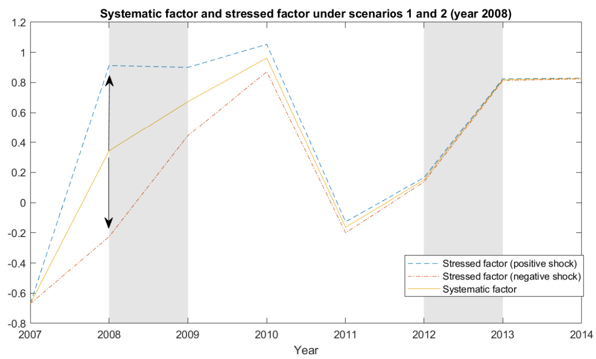

Figure 13: Systematic Factor and Stresses Factor under Scenarios 1 and 2 (year 2008). This Figure exhibits the systematic factor and the stressed factor under both scenarios, shocks are applied in 2008. The shaded areas indicate the OECD recessions in France.

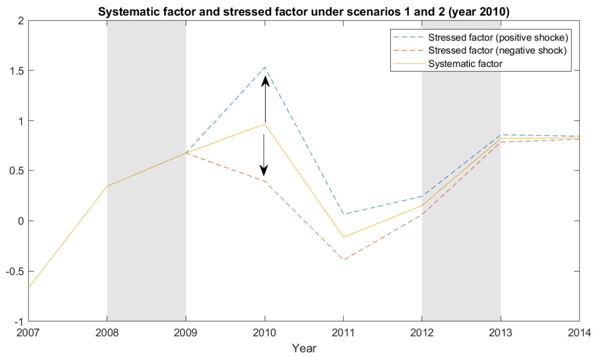

Figure 14: Systematic Factor and Stresses Factor under Scenarios 1 and 2 (year 2010). This Figure exhibits the systematic factor and the stressed factor under both scenarios, shocks are applied in 2010. The shaded areas indicate the OECD recessions in France.

When we impose the two shocks to the systematic factor in 2008, the effects of shocks are significant for the next two periods. Afterward, the effects dissipate gradually. We observe the same pattern when we impose the shocks to the systematic factor in 2010. The shocks seem to dissipate faster when applied in 2010, i.e. the economy after a recession.

Let us compare the total default rates for all rating classes. We compute the stressed migration matrices from 2008 to 2014 and compare the estimated total default rates by all rating categories and stressed total default rates by all rating classes in the years 2008 to 2014. Next, we compare the stability rates, downgrade and upgrade probabilities. Figure 15 displays the estimated and stressed total default rates when we impose positive or negative shocks to the systematic factor in 2008.

Once we impose the positive shock to the factor, the total default rates of all rating classes decrease over the years 2008 to 2013. We observe a 0.61% reduction in the total default rates in 2008 and 0.2% in 2010. The effect of positive shock is stronger in 2008 than in 2009. Afterward, the total default rates are decreasing until 2011 and the effects of shock disappear. Then, the shocking default rates go back to estimated levels of default rates.

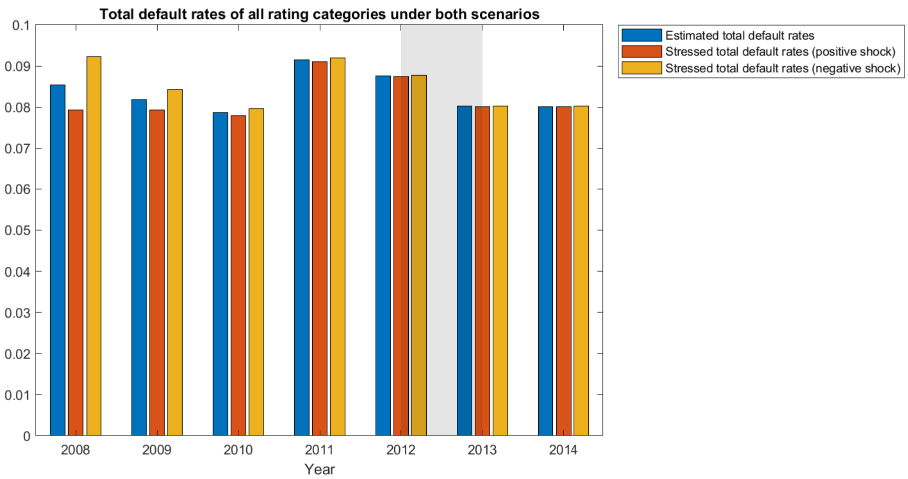

Figure 15: Total Default Rates of all Rating Classes under both Scenarios (2008). This Figure shows the estimated total default rates of all rating categories (blue) for years 2008 to 2014, the stressed total default rates after positive shock (red) and the stressed total default rates after negative shock (yellow). The shaded areas indicate the OECD recessions in France.

As a consequence of the one-time negative shock to the systematic factor in the year 2008, the total default rates of all rating categories increase over the period 2008 to 2013. We observe an increase of 0.68% in 2008 and of 0.25% in 2009. The effect of the negative shock is stronger in 2008 than in 2009. Afterward, the total default rates keep increasing until 2011. Then, the effects of shock disappear and the shocked default rates go back to the estimated levels of default rates. Next, let us compare the downgrade rates. Table 16 shows the changes in downgrade rates for each rating category over the years 2008 to 2014 under both positive and negative shocks to the systematic factor in 2008.

The downgrade rates for all rating classes decrease when we impose the positive shock to the factor and increase with the one-time negative shock to the systematic factor in 2008 except for firms rated "D". For instance, the downgrade rates of firms rated "A+" have decreased by 1.8% at the time we imposed the positive shock. They increased by almost 2% when we impose a negative shock to the factor. The effect of the positive and negative shocks are much stronger in the first two years after the shocks as compared to other years.

Table 16: The Changes in Downgrade Rates

Rating class	**changes in the downgrade rates**						
	One-time positive shock to the factor in year 2008						
	2008	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**
A+	-0.0180	-0.0071	-0.0028	-0.0012	-0.0005	-0.0002	-0.0001
A	-0.0264	-0.0106	-0.0043	-0.0016	-0.0007	-0.0003	-0.0001
B+	-0.0143	-0.0057	-0.0023	-0.0009	-0.0004	-0.0001	-0.0001
B	-0.1848	-0.0666	0.0219	-0.0144	-0.0057	-0.0017	-0.0007
C	-0.0531	-0.0211	-0.0083	-0.0035	-0.0014	-0.0005	-0.0002
D	0.00044	0.00017	7.11E-05	2.78E-05	1.12E-05	4.53E-06	1.81E-06
	One-time negative shock to the factor in year 2008						
A+	0.0189	0.0072	0.0028	0.0012	0.0005	0.0002	0.0001
A	0.0258	0.0105	0.0043	0.0016	0.0007	0.0003	0.0001
B+	0.0141	0.0057	0.0023	0.0009	0.0004	0.0001	0.0001
B	0.2236	0.0775	0.0239	0.0143	0.0058	0.0017	0.0007
C	0.0545	0.0214	0.0083	0.0035	0.0014	0.0005	0.0002
D	-0.00044	-0.00018	-7.10E-05	-2.78E-05	-1.12E-05	-4.53E-06	-1.81E-06

The one-time positive and negative shocks to the systematic factor have more effect on firms rated "B" at the time the shocks are imposed.

Next, we impose the same shocks to the systematic factor in the year 2010 which is a period after the financial crisis. We analyze the effect of shocks on the total default rates of all rating classes, and on the downgrade rates. Figure 16 shows the estimated total default rates and the stressed total default rates of all rating categories under both scenarios when we impose the shocks in the year 2010.

The total default rates of all rating categories decrease by 0.54% in 2010, 0.28% in 2011 and 0.11% in 2012. Over the years 2013 and 2014, we observe less decrease in total default rates of all rating categories.

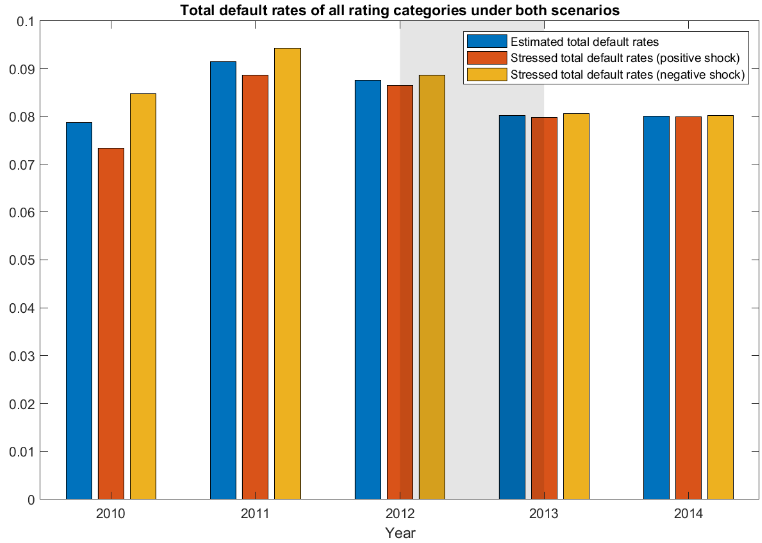

Figure 16: Total Default Rates of all Rating Classes under both Scenarios (2010). This Figure shows the estimated total default rates of all rating categories (blue) for the years 2010 to 2014, the stressed total default rates with positive shock (red) and the stressed total default rates with negative shock (yellow). The shaded areas indicate the OECD recessions in France.

Moreover, we observe an increase in the total default rates of all rating categories when we impose a negative shock to the factor. Specifically, the total default rates increase by 0.61%, 0.29% and 0.11% over the years 2010 to 2012, respectively. We also have increases of 0.04% and of 0.02% in total default rates for the years 2013 and 2014. The effect of the negative shock similar to the positive shock is stronger in the first two years after the shock. Afterward, the effect of shock diminishes.

The changes in downgrade rates under the positive or negative one-time shock to the systematic factor in 2010 are given in Table 17. The results of a one-time negative shock to the systematic factor in 2010, are similar to those of a one-time negative shock in 2008. However, the size of the effects is different. The effects of one-time positive and negative shocks in the year 2008 are larger since the economy experiences recession over the period 2008 to 2009, while in the year 2010, there is no recession.

Table 17: The Changes in Downgrade Rates

Rating class	changes in the downgrade rates				
	One-time positive shock to the factor in year 2010				
	2010	**2011**	**2012**	**2013**	**2014**
A+	-0.0170	-0.0076	-0.0030	-0.0011	-0.0005
A	-0.0269	-0.0103	-0.0042	-0.0017	-0.0007
B+	-0.0144	-0.0056	-0.0023	-0.0009	-0.0004
B	-0.1051	-0.0905	-0.0356	-0.0102	-0.0041
C	-0.0507	-0.0219	-0.0087	-0.0034	-0.0013
D	0.00044	0.00017	6.99E-05	2.83E-05	1.13E-05
	One-time negative shock to the factor in year 2010				
A+	0.0179	0.0078	0.0030	0.0011	0.0005
A	0.0264	0.0102	0.0041	0.0017	0.0007
B+	0.0143	0.0056	0.0023	0.0009	0.0004
B	0.1791	0.0572	0.0361	0.1050	0.0042
C	0.0530	0.0219	0.0087	0.0034	0.0013
D	-0.00044	-0.00017	-6.98E-05	-2.83E-05	-1.13E-05

We observe a decline in downgrade rates for all the rating classes except for firms rated "D" in scenario 1 (positive shock) and vice versa in scenario 2 (negative shock). The next section analyzes the effects of one-time positive and negative shocks to the systematic factor under the Ceteris Paribus assumption.

7.2 Shocks to the Systematic Factor- Assuming "Ceteris Paribus"

Under the Ceteris Paribus assumption, the factor has a flat baseline to which it returns after the shocks. We examine the effect of shocks to observe how quickly the shocks and their effects dissipate. The one-time positive and negative shocks are introduced in the year 2008, which is during a financial crisis. Alternatively, the shocks are imposed in the year 2010 after the recession. Figures 17 and 18 show the paths of the systematic factor and of the stressed factor under both stress scenarios in the years 2008 and 2010 respectively under the Ceteris Paribus assumption.

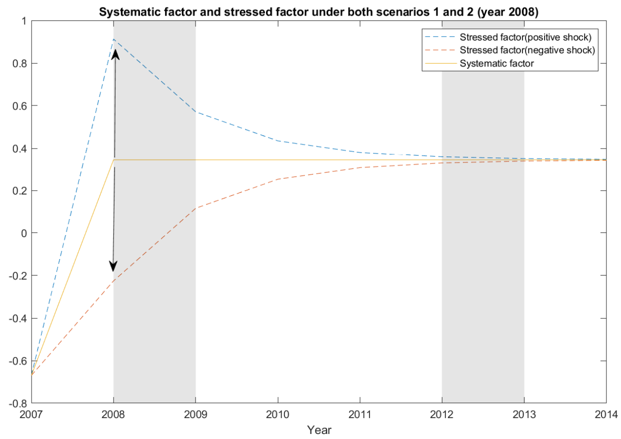

Figure 17: Systematic Factor and Stresses Factor under Scenarios 1 and 2 (year 2008). This Figure exhibits the systematic factor and the stressed factor under both scenarios, Ceteris Paribus, shocks are applied in 2008. The shaded areas indicate the OECD recessions in France.

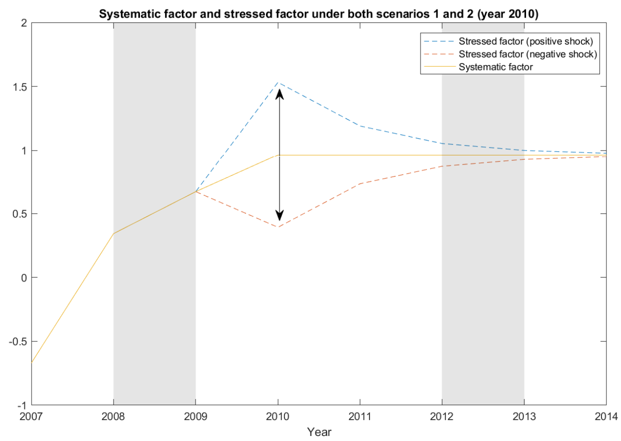

Figure 18: Systematic Factor and Stresses Factor under Scenarios 1 and 2 (year 2010). This Figure exhibits the systematic factor and the stressed factor under both scenarios, Ceteris Paribus, shocks are applied in 2010. The shaded areas indicate the OECD recessions in France.

From the graph, we observe that the effects of positive and negative shocks to the systematic factor during the financial crisis of 2008, are higher than the effects of shocks in 2010. Moreover, these effects are much stronger in the first 2 years after the shocks to the systematic factor in both years 2008 and 2010.

Next, let us compare the total default rates of all rating categories under both scenarios when we impose the shocks to the systematic factor in 2008, during the recession. The total default rates of all rating classes under both stress scenarios are shown in Figure 19 for the year 2008 and Figure 20 for the year 2010.

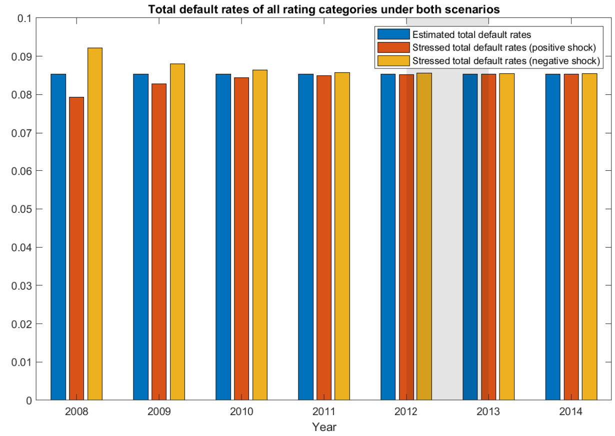

Figure 19: Total Default Rates of all Rating Classes under both Scenarios (2008). This Figure shows the estimated total default rates of all rating categories (blue) for years 2008 to 2014, the stressed total default rates after positive shock (red) and the stressed total default rates after negative shock (yellow). The shaded areas indicate the OECD recessions in France.

The total default rates decrease after a one-time positive shock to the systematic factor in the year 2008. In 2008, the total default rates have decreased by 0.61% after the one-time positive shock to the factor. They have been increased by 0.68% after the one-time negative shock. We observe similar dissipation patterns of both shocks in the following periods. The size of effects diminishes over time. The effect of a one-time negative shock is slightly higher than of one-time positive shock to the factor.

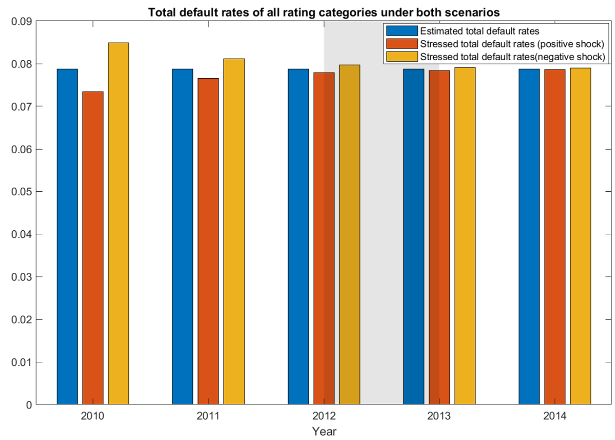

Figure 20: Total Default Rates of all Rating Classes under both Scenarios (2010). This Figure shows the estimated total default rates of all rating categories (blue) for years 2010 to 2014, the stressed total default rates after positive shock (red) and the stressed total default rates after negative shock (yellow). The shaded areas indicate the OECD recessions in France.

When the shocks are imposed on the systematic factor in the years 2008, and 2010, we observe that the total defaults of all rating classes decrease after the positive shock and increase after the negative shock to the factor. The effects of shocks are stronger over the years 2010 until 2012 and diminish later on. The total default rates of all rating categories decrease by 0.54% after the positive shock and increase by 0.61% after the negative shock to the factor in 2010. This is less than the decrease (increase) observed when we imposed shocks during the recession in 2008.

Next, we explore the effects of shocks on the downgrade rates. Table 18 displays the changes in the downgrade rates for all rating categories when the positive and negative shocks are imposed on the systematic factor in the year 2008. The one-time positive shock to the factor reduces the downgrade rates of all firms except for firms rated "D". However, the change in the rating category "D" is very small. In contrast, the one-time positive shock to the factor increases the downgrade rates of all firms except for firms rated "D".

Table 18: The Changes in the Downgrade Rates

Rating class	**changes in the downgrade rates**						
	One-time positive shock to the factor in year 2008						
	2008	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**
A+	-0.0180	-0.0073	-0.0029	-0.0012	-0.0005	-0.0002	-0.0001
A	-0.0264	-0.0105	-0.0042	-0.0017	-0.0007	-0.0003	-0.0001
B+	-0.0143	-0.0057	-0.0023	-0.0009	-0.0004	-0.0001	-0.0001
B	-0.1848	-0.0816	-0.0337	-0.0136	-0.0055	-0.0022	-0.0009
C	-0.0531	-0.0215	-0.0086	-0.0035	-0.0014	-0.0006	-0.0002
D	0.000441	0.00018	7.02E-05	2.81E-05	1.12E-05	4.49E-06	1.80E-06
	One-time negative shock to the factor in year 2008						
A+	0.0189	0.0074	0.0030	0.0012	0.0005	0.0002	0.0001
A	0.0258	0.0104	0.0042	0.0017	0.0007	0.0003	0.0001
B+	0.0141	0.0057	0.0023	0.0009	0.0004	0.0001	0.0001
B	0.2236	0.0882	0.0347	0.0138	0.0055	0.0022	0.0009
C	0.0545	0.0217	0.0086	0.0035	0.0014	0.0006	0.0002
D	-0.000436	-0.00017	-7.01E-05	-2.80E-05	-1.12E-05	-4.49E-06	-1.80E-06

We observe similar results when the shocks are imposed on the factor in the year 2010. The effects of shocks are slightly smaller than when the shocks are imposed on the factor during the financial crisis. These shocks have more effects on the firms rated "B" and "C".

The importance of assumption Ceteris Paribus, or "other things being equal or held constant", is to determine the causation. Once, we fix the factor, we are able to describe the effects of shocks on the migration probabilities only. Under both "Ceteris Paribus" and "Mutatis Mutandis" assumptions, the one-time positive shock to the factor in both years 2008 and 2010, causes less default and downgrades for the majority of firms and a one-time negative shock increases the default and downgrades of firms.

Table 19: The Changes in the Downgrade Rates

Rating class	changes in the downgrade rates				
	One-time positive shock to the factor in year 2010				
	2010	**2011**	**2012**	**2013**	**2014**
A+	-0.0170	-0.0069	-0.0028	-0.0011	-0.0004
A	-0.0269	-0.0107	-0.0043	-0.0017	-0.0007
B+	-0.0144	-0.0058	-0.0023	-0.0009	-0.0004
B	-0.1051	-0.0509	-0.0219	-0.0090	-0.0036
C	-0.0507	-0.0206	-0.0083	-0.0033	-0.0013
D	0.0004	0.0001	7.11E-05	2.84E-05	0.0000
	One-time negative shock to the factor in year 2010				
A+	0.0179	0.0071	0.0028	0.0011	0.0004
A	0.0264	0.0106	0.0043	0.0017	0.0007
B+	0.0143	0.0057	0.0023	0.0009	0.0004
B	0.1791	0.0633	0.0239	0.0093	0.0037
C	0.0530	0.0210	0.0083	0.0033	0.0013
D	-0.00044	-0.00018	-7.10E-05	-2.84E-05	0.0000

In our stress-testing analysis, the effects of shocks to the factor are slightly higher when we assume the Ceteris Paribus assumption especially, for the firms rated "B" and "C". Under both assumptions and both stress scenarios, we observe higher rates of default and downgrades for firms after the negative shock to the factor and lower rates of default and downgrades for firms after the positive shock to the factor.

The regression analysis in Section 5.2.1 and the stress-testing analysis help the interpretation of the unobserved factor in the corporate credit migration rates. As discussed in Section 5.2.1, the unobserved factor seems to be close to the dynamics of several leading macro indicators, such as the output gap and the total industrial production variables and the short-term interest rate. Moreover, we found graphically similar patterns between the unobserved factor and the annual return on the CAC 40 index in France. However, the factor interpretation is finalized following the stress testing analysis.

Based on the first stress scenario, a one-time positive shock to the factor reduces the default rates and the downgrade rates of firms. After the one-time positive shock, the economy experiences an upturn. Therefore, we observe fewer default and downgrades and higher stability and upgrade rates. Alternatively, based on the second stress scenario, a one-time negative shock to the factor, causes more default and downgrades firms since the economy experiences a downturn. Because the factor is negatively correlated with the output gap, the total industrial production and the short-term interest rate, a positive shock to any of these variables causes to decrease in the factor and a negative shock increases the factor.

Therefore, the unobserved factor could not be interpreted as the output gap or the total industrial production in France, since, the one-time positive shock to these two variables decreases the factor. On the other hand, the unobserved factor could be interpreted as the short-term interest rate, due to the fact that once banks decrease their interest rates, the cost of borrowing from the banks decrease and as a result, firms have less default and downgrade rates. Normally, we expect to see a decrease in the interest rates during the financial crisis or economic downturn as a monetary policy impose by central banks to offset the normal forces of credit supply and demand for liquidity. During the financial crisis in 2008, the short term interest rate in France decreases by 0.32%, however, our stress test analysis, shows that this magnitude of the decrease is not quite significant to prevent the rise in the downgrade rates and default rates of firms in France. The results of our stress test analysis show that a shock either positive or negative with the size of one standard deviation of the estimated factor (0.56) has a significant impact on the firm's downgrade rates and default probabilities in France. The size of the shock is about the one third of the standard deviation of the short-term interest rate in France, which is 1.71 (Table 38, Appendix C) in our sample period. Therefore, our results shows that during the economic downturn like the financial crisis in 2008, the short-term interest rate in France has to fell at least by its two-third of its own standard deviation in our sample period, to prevent the rise in the migration probabilities, specifically, the downgrade rates and default probabilities.

Moreover, the systematic factor can be interpreted as the annual return on CAC 40 which represents the overall level and direction of the market in France, and the factor has a similar pattern and positive correlation with it. An increase in stock market indices improves the financial conditions of firms. They are also a leading indicator of the future economy and reflect the expectation of future profits. Due to the results of the stress-testing analysis, it seems that the unobserved factor could be interpreted as the short-term interest rate or the annual return on the CAC 40 index in France.

8 Concluding Remarks

We use the stochastic factor Probit migration model which is a dynamic (non-linear) risk factor model, to study the dynamics of credit rating matrices by introducing an unobserved (latent) common factor that represents a fundamental driving process. This model is an important tool for credit rating and default probability estimation, as pointed out in the Basel II report on Credit Risk Factor Modeling. The stochastic factor Probit model has been estimated on the French dataset of a European bank by two estimation methodologies. First, the stochastic factor Probit model has been written as an approximate linear state-space model and estimated by means of the standard Kalman filter. The unobserved factor has been obtained exogenously by principal component analysis in this approach. Second, the factor Probit model has been estimated by the two-step efficient estimation approach. The estimates of the approximate linear state-space model have been used as initial values in the numerical algorithm to maximize the log-likelihoods in the two-step efficient estimation. In the two-step approach, the factor values have considered as nuisance parameters and the estimator of the micro-component was a fixed effects estimator. In the second step of the two-step procedure, the unobserved factor values have been replaced by the cross-sectional factor approximations and the estimator of the macro-component has been obtained by applying maximum likelihood (ML) on the autoregressive AR(1) model. We assessed the stochastic migration model on its ability to link the transition probabilities to an unobserved dynamic risk factor. Our findings shows that the factor Probit model fits well with the dataset. The implementation of the two-step efficient estimation methodology is easy and less time consuming. The estimates of the factor Probit model parameters obtained by the approximated linear state-space model have been improved by the two-step estimator. The unobserved factor has been recovered by the two-step estimation method and it seems to be close to the dynamics of several leading macro indicators, such as the output gap and the total industrial production variables from the national account group and the short-term interest rate from the financial markets group. Moreover, we found graphically a similar pattern between the unobserved factor and the annual return on the CAC 40 index in France. However, the factor interpretation is carried out by using a stress testing analysis. We examined the effects of one-time shocks to the factor on credit migrations and default probabilities. The shocks are used to evaluate stressed migration probabilities and default probabilities during and after crisis of 2008. Our analysis reveals that the shock effects help determine without ambiguity which macroeconomic variable has the same characteristics as the factor and can be interpreted as such. This finding has implications for policy making during economic downturns, such as crisis of 2008 or Covid-19 pandemic. The stress-testing results allowed us to interpret the factor, which points to the short-term interest rate in France. We observed a noticeable decrease in the total default rates of all rating categories in the year we introduced a one-time positive shock to the factor and increase in the total default rates of all rating categories when we introduced a one-time negative shock to the factor. The stress scenarios are applied to the economy during the downturn of 2008 and after the recovery in 2010. We observed that the shock effects differ and are more apparent in the post-crisis economy. During the financial crisis in 2008, the short term interest rate in France decreases by 0.32%, however, our stress test analysis, shows that this magnitude of the decrease is not quite significant to prevent the rise in the downgrade rates and default rates of firms in France. The results of our stress test analysis show that a shock either positive or negative with the size of one standard deviation of the estimated factor (0.56) has a significant impact on the firm's downgrade rates and default probabilities in France. The size of the shock is about the one third of the standard deviation of the short-term interest rate in France, which is 1.71 in our sample period. Accordingly, our results shows that during the economic downturn like the financial crisis in 2008, or the Covid-19 pandemic, the short-term interest rate in France has to fall more significantly at least by its two-third of its own standard deviation in our sample period, to prevent the rise in the migration probabilities, specifically, the downgrade rates and default probabilities. Thus, the model can be used for macro-stress testing purposes.

Appendices

Appendix A

An illustration of the approximate method in section (2.2.1) is outlined in Section. We consider a stochastic migration model with three $K = 3$ rating categories $k = 1, ..., 3$. The individual rating $y_{i,t}$ of firm $i = 1, ..., n$ can be in either qualitative rating class "1", "2", or "3" at time $t = 1, ..., T$. The class "1" corresponds to the high-risk category of default and "3" refers to the lowest risk category. There is a pool of homogeneous corporates. Each of them has a credit rating at time $t-1$ and by the time t, either one of three transitions can occur:

- First: Firm i stays in the same rating class
- Second: Firm i is upgraded to a higher rating class
- Third: Firm i is downgraded to a lower rating class

Table 2.1 provides the counts of firms in each class, which is the main summary that can be used to calculate the transition frequencies for each rating category as follows:

Table 20: Number of firms for each class

(t-1)	(t)			
	3	2	1	Issuers
3	$n_{3,3,t}$	$n_{3,2,t}$	$n_{3,1,t}$	$n_{3,t-1}$
2	$n_{2,3,t}$	$n_{2,2,t}$	$n_{2,1,t}$	$n_{2,t-1}$
1	$n_{1,3,t}$	$n_{1,2,t}$	$n_{1,1,t}$	$n_{1,t-1}$
Total	$n_{3,t}$	$n_{2,t}$	$n_{1,t}$	

Thus, the transition matrix can be calculated as follows:

Table 21: Transition matrix for year t. $\hat{p}_{l,k,t}$ is observed transition frequency from category l to k at time t.

	3	2	1	Sum
3	$(n_{3,3,t})/(n_{3,t-1})$=$p_{3,3,t}$	$(n_{3,2,t})/(n_{3,t-1})$=$p_{3,2,t}$	$(n_{3,1,t})/(n_{3,t-1})$=$p_{3,1,t}$	1
2	$(n_{2,3,t})/(n_{2,t-1})$=$p_{2,3,t}$	$(n_{2,2,t})/(n_{2,t-1})$=$p_{2,2,t}$	$(n_{2,1,t})/(n_{2,t-1})$=$p_{2,1,t}$	1
1	$(n_{1,3,t})/(n_{1,t-1})$=$p_{1,3,t}$	$(n_{1,2,t})/(n_{1,t-1})$=$p_{1,2,t}$	$(n_{1,1,t})/(n_{1,t-1})$=$p_{1,1,t}$	1

The transition matrices $\hat{P}_t = [p_{l,k}]_t$ are observed at each time t. Their dimensions are $(K \times K)$ and each P_t has K^2 elements, $k = 1, ..., k$. The variation of theses matrices over time is due to the latent factor f_t. The firm i gets the rating in class k, where $k = 1, 2, 3$ if the latent score y^*_{it} lies between threshold $c's$

$$y_{i,t} = k \text{ if } c_{k-1} < y^*_{i,t} < c_k$$

Since there are three rating categories, we have two thresholds, denoted by c_1 and c_2. Thus we have:

$$
\begin{aligned}
y_{i,t} &= 1 \ \text{ if } y^*_{i,t} \leq c_1 \\
y_{i,t} &= 2 \ \text{ if } c_1 < y^*_{i,t} \leq c_2 \\
y_{i,t} &= 3 \ \text{ if } y^*_{i,t} > c_2
\end{aligned}
$$

By assuming that the latent scores are linear functions of factor f_t, the linear factor model can be written as follows:

$$
\begin{aligned}
y_{i,t-1} = 1 &\rightarrow y^*_{i,t} = \delta_1 + \beta_1 f_t + \sigma_1 u_{i,t} \\
y_{i,t-1} = 2 &\rightarrow y^*_{i,t} = \delta_2 + \beta_2 f_t + \sigma_2 u_{i,t} \\
y_{i,t-1} = 3 &\rightarrow y^*_{i,t} = \delta_3 + \beta_3 f_t + \sigma_3 u_{i,t}
\end{aligned}
$$

where the factor f_t satisfy a Gaussian Autoregressive (AR(1)) process. The error terms $u_{i,t}$ are i.i.d normal with mean zero and variance one. There are 9 unknown parameters ($\delta_1, \delta_2, \delta_3, \beta_1, \beta_2, \beta_3, \sigma_1, \sigma_2$, and σ_3) and two thresholds (c_1, c_2) and T factor values $(f_1, ...f_T)$. The Probability of staying in category "3" is:

$$
\begin{aligned}
p_{3,3,t} &= P[y^*_{i,t} > c_2 | f_t, y_{i,t-1} = 3], \\
&= P[\delta_3 + \beta_3 f_t + \sigma_3 u_{i,t} > c_2], \\
&= P[u_{i,t} > \frac{c_2 - \delta_3 - \beta_3 f_t}{\sigma_3}], \\
&= 1 - \Phi(\frac{c_2 - \delta_3 - \beta_3 f_t}{\sigma_3}),
\end{aligned}
$$

The transition probabilities from category 3 to 2 is characterized by:

$$
\begin{aligned}
p_{3,2,t} &= P[c_1 < y^*_{i,t} \leq c_2 | f_t, y_{i,t-1} = 3], \\
&= \Phi(\frac{c_2 - \delta_3 - \beta_3 f_t}{\sigma_3}) - \Phi(\frac{c_1 - \delta_3 - \beta_3 f_t}{\sigma_3}),
\end{aligned}
$$

And lastly, the transition from category 3 to 1:

$$
\begin{aligned}
p_{3,1,t} &= P[y^*_{i,t} \leq c_1 | f_t, y_{i,t-1} = 3], \\
&= \Phi(\frac{c_1 - \delta_3 - \beta_3 f_t}{\sigma_3}),
\end{aligned}
$$

Each row of the transition matrices is an ordered Probit as the 3 transition probabilities are linked due to the presence of the thresholds c_1 and c_2. The ratios $(\frac{c_k - \delta_l - \beta_l F_t}{\sigma_l})$ in the above transition probabilities identify semi-parametrically the micro-parameters and the factor values up to location and scale transformations. There are identification problems due to the factor dynamics and the partial observability of the quantitative score $y^*_{i,t}$. As stated before in this approach we consider two identification constraints for the factor dynamics and two identification restrictions for the micro-parameters. [See, Gagliardini and Gouriéroux (2005) for the complete discussion on the identification restrictions on the stochastic migration model] The first identification constraint is about the factor dynamics. Since, f_T is unobservable, and has to be defined up to an invertible linear transformation either additive or multiplicative:

$$f_t \rightarrow f_t + constant$$

$$f_t \to \lambda f_t$$

Therefore, two identification constraints can be imposed for factor dynamics f_t:

$$E(f_t) = 0$$

$$V(f_t) = 1$$

The second identifiability problem is due to the partial observability of the quantitative score. Since the same combination of affine transformations of score and threshold can give the same migration probabilities, two standard identifications $c_1 = 0$ and $\sigma_1^2 = 1$ are needed to be imposed. These constraints have to imposed to link rows of transition matrix and to allow for differences between the micro- and macro parameters.

From the data in migration matrices, the values of $p_{3,3,t}$, $p_{3,2,t}$ and so on, are observed. Under the granularity approach, it is assumed that as $n \to \infty$ we have $\hat{p}_{l,k,t} \approx p_{l,k,t}$. Therefore we can write:

$$\hat{p}_{l,k,t} = \Phi^{-1}(p_{l,k,t}) = \frac{c_k - \delta_l - \beta_l f_t}{\sigma_l} \text{ for } l = 1,2,3 \text{ and } k = 1,2,3$$

Now the model can be re-parameterized in the appropriate way to obtain the canonical factors $a's$ as follows:

- First row:

$$a_{3,3,t} = \Phi^{-1}(p_{3,3,t}) = \frac{c_2-\delta_3-\beta_3 f_t}{\sigma_3}$$
$$a_{3,2,t} = \Phi^{-1}(p_{3,2,t} + p_{3,3,t}) = \frac{c_1-\delta_3-\beta_3 f_t}{\sigma_3}$$

- Second row:

$$a_{2,3,t} = \Phi^{-1}(p_{2,3,t}) = \frac{c_2-\delta_2-\beta_2 f_t}{\sigma_2}$$
$$a_{2,2,t} = \Phi^{-1}(p_{2,2,t} + p_{2,3,t}) = \frac{c_1-\delta_2-\beta_2 f_t}{\sigma_2}$$

- Third row:

$$a_{1,3,t} = \Phi^{-1}(p_{1,3,t}) = \frac{c_2-\delta_1-\beta_1 f_t}{\sigma_1}$$
$$a_{1,2,t} = \Phi^{-1}(p_{1,2,t} + p_{1,3,t}) = \frac{c_1-\delta_1-\beta_1 f_t}{\sigma_1}$$

There are 6 approximately affine transformation of F_t, which can be written as follows:

$$a_{l,k,t} = \hat{p}_{l,k,t} = \Phi^{-1}(p_{l,k,t}) \approx \frac{c_k-\delta_l}{\sigma_l} - \frac{\beta_l}{\sigma_l} f_t$$

There are two alternatives ways to define the representation of factors:

- One approach is as follows: Since factors are affine linear transformation, we can choose one of the canonical factors (one cell in the transition matrix) and consider it as the common factor. In order to impose identification restrictions on the factor dynamics, we can demean and standardize it. Then, we get the following factor representation:

$$\hat{f}_t = \frac{a_{3,3,t} - \frac{1}{T}\sum_{t=1}^{T} a_{3,3,t}}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(a_{3,3,t} - \bar{a}_{3,3})^2}}$$

- An alternative approach is the following: We determine the number of factors first, by using the principal component analysis (PCA), which is the spectral decomposition of the $(T * T)$ matrix (YY'). This matrix contains Â the series of estimated canonical factors $\hat{a}_{l,k,t}$ where each row of that matrix is $(\hat{a}_{l,k,t} - \bar{a}_{l,k})$, where, $(\bar{a}_{l,k} = \frac{1}{T}\sum_{t=1}^{T} a_{l,k,t})$. Then, we need to find the eigenvalues and eigenvectors associated with these eigenvalues. Next, we find the number of factors form the number of the largest eigenvalues.Â

Under the second approach, we can impose the identification constraints on the factor dynamics by demeaning and standardizing the largest eigenvalues or the associated eigenvectors. In this case, we need fewer constraints on the micro-parameters. Below we consider the second representation of factors and two restrictions on factor dynamics and micro-parameters will be imposed.

Due to the identification restrictions on factor dynamics, and given $E(f_t) = 0$, we can identify:

$$\frac{c_k - \delta_l}{\sigma_l} = \frac{1}{T}\sum_{t=1}^{T} p_{l,k,t}$$

$$\frac{\beta_l}{\sigma_l} = \text{Standard error of } \hat{p}_{l,k,t}$$

Next given the other identification restrictions on δ and σ, we can consistently estimate all the parameters. The complete derivation of the asymptotic properties of the estimators can be found in Gagliardini and Gourieroux (2014), which is a comprehensive study of granularity theory in finance and insurance.

Appendix B

The original adjusted migration probabilities for the period of 2007-2014 are reported in Appendix B.1 and the adjusted migration probabilities for the period of 2007-2014 are reported in B.2.

Appendix B.1

The original adjusted migration probabilities for the period of 2007-2014 are reported in Table 22-29

Table 22: The original adjusted transition matrix for year 2007

2007	ISSUER	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	91	53.54	13.13	13.13	10.10	3.03	2.02	4.04	0.00	0.00	0.00	1.01	0.00	0.00	0.00
2	322	2.80	27.33	24.53	11.80	7.76	9.63	3.42	5.59	3.42	1.24	0.31	1.24	0.62	0.31
3	1132	0.88	2.39	28.62	11.93	13.34	16.43	9.36	6.45	4.51	3.36	1.41	0.62	0.62	0.09
4	3181	0.03	0.16	2.77	23.45	14.40	19.96	15.72	11.76	5.94	2.92	1.54	0.53	0.50	0.31
5	2904	0.07	0.03	2.07	4.75	34.92	13.50	15.50	12.36	8.54	3.99	2.10	0.93	0.55	0.69
6	8120	0.01	0.10	0.83	1.98	7.46	29.31	17.40	19.47	12.40	5.99	2.46	0.92	0.84	0.83
7	7263	0.04	0.10	0.54	0.81	3.95	6.83	36.09	18.24	18.08	9.14	3.37	1.25	0.77	0.78
8	6252	0.03	0.03	0.22	0.75	2.91	5.52	11.72	42.25	17.93	11.28	3.82	1.60	0.86	1.07
9	14256	0.02	0.04	0.14	0.32	1.36	3.31	9.48	13.73	38.53	20.19	6.76	2.62	1.98	1.52
10	9207	0.01	0.02	0.09	0.24	1.08	1.80	6.10	9.51	15.90	43.04	10.49	4.95	3.65	3.11
11	3356	0.03	0.00	0.12	0.06	0.60	1.40	3.81	5.90	11.86	19.85	34.51	9.71	6.94	5.21
12	2641	0.04	0.00	0.08	0.38	0.27	1.10	3.10	5.26	9.01	16.74	13.86	31.58	11.66	6.93
13	2451	0.08	0.12	0.00	0.16	0.69	1.43	1.55	3.59	6.00	9.63	9.47	9.71	48.92	8.65
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100.00

Table 23: The original adjusted transition matrix for year 2008

2008	ISSUER	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	63	79.00	3.00	2.00	0.00	14.00	0.00	2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	155	2.58	53.55	12.26	9.68	5.81	4.52	3.87	3.23	1.29	1.29	0.00	0.65	1.29	0.00
3	727	0.28	2.61	53.37	18.16	9.22	5.09	3.30	3.16	1.93	0.96	0.96	0.14	0.41	0.41
4	1547	0.06	1.16	5.17	47.32	15.06	12.73	6.08	4.27	3.62	2.00	0.65	0.45	0.65	0.78
5	3504	0.06	0.17	1.43	4.14	52.94	14.27	10.45	5.88	4.79	2.60	1.14	0.43	0.66	1.06
6	5920	0.02	0.03	0.44	1.99	6.72	48.87	18.26	10.29	5.84	4.14	1.30	0.52	0.79	0.78
7	9304	0.02	0.05	0.26	0.56	2.69	7.93	52.43	15.94	9.69	5.30	2.16	0.87	0.76	1.34
8	11695	0.00	0.07	0.13	0.37	1.28	3.67	10.69	50.86	17.29	8.85	3.04	1.10	1.18	1.48
9	13990	0.03	0.05	0.05	0.29	0.79	2.00	5.70	14.20	48.08	16.64	5.83	2.43	2.07	1.85
10	12400	0.00	0.05	0.07	0.10	0.48	1.02	2.69	6.56	16.78	49.09	11.23	5.02	3.20	3.70
11	5401	0.06	0.11	0.02	0.15	0.43	0.94	2.26	4.68	9.65	18.76	40.62	8.79	6.92	6.61
12	3127	0.00	0.03	0.03	0.13	0.26	0.54	1.31	3.42	6.81	12.18	14.58	41.64	12.12	6.94
13	3420	0.03	0.15	0.00	0.09	0.23	0.67	1.32	2.69	4.06	6.70	7.49	8.45	58.22	9.91
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100.00

Table 24: The original adjusted transition matrix for year 2009

2009	ISSUER	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	35	68.32	0.00	10.89	2.97	0.00	0.00	8.91	2.97	2.97	2.97	0.00	0.00	0.00	0.00
2	130	0.77	64.62	16.92	8.46	4.62	3.08	1.54	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	622	0.48	6.59	61.26	8.84	8.84	2.89	4.34	3.38	1.77	0.32	0.48	0.32	0.32	0.16
4	1340	0.00	0.52	10.82	52.40	15.90	7.76	5.22	2.46	1.94	1.64	0.37	0.30	0.52	0.15
5	3310	0.00	0.18	2.02	8.16	52.91	11.66	10.24	6.47	4.26	2.54	0.60	0.18	0.36	0.42
6	5506	0.02	0.13	0.73	2.56	9.04	50.22	16.75	9.10	5.58	3.20	0.96	0.40	0.80	0.53
7	9427	0.00	0.08	0.19	0.66	3.17	8.78	50.51	16.54	10.31	5.42	2.28	0.69	0.91	0.46
8	12233	0.01	0.10	0.12	0.36	1.28	3.26	10.98	50.37	17.46	9.13	3.45	1.33	1.14	1.01
9	14137	0.02	0.09	0.07	0.21	0.95	1.73	5.37	13.32	48.53	17.29	6.47	2.50	2.00	1.45
10	12988	0.00	0.15	0.09	0.18	0.41	0.85	2.66	5.57	15.77	50.81	11.60	5.19	3.84	2.88
11	6204	0.00	0.03	0.03	0.08	0.44	0.55	2.42	3.74	8.61	18.05	41.68	11.19	8.01	5.17
12	3674	0.03	0.11	0.05	0.05	0.19	0.52	1.69	3.29	5.09	11.35	13.94	40.88	15.13	7.68
13	4144	0.02	0.07	0.05	0.00	0.27	0.39	1.54	2.05	3.26	5.91	6.13	7.00	63.39	9.92
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100.00

Table 25: The original adjusted transition matrix for year 2010

2010	ISSUER	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	29	81.44	3.09	3.09	0.00	3.09	3.09	3.09	3.09	0.00	0.00	0.00	0.00	0.00	0.00
2	230	0.00	85.23	6.52	3.91	0.00	2.17	0.87	0.00	0.87	0.43	0.00	0.00	0.00	0.00
3	706	0.28	4.39	62.75	15.16	8.78	4.11	0.99	0.57	1.56	0.71	0.00	0.28	0.28	0.14
4	1329	0.08	0.68	8.13	57.49	11.96	11.89	4.89	1.88	1.35	1.05	0.45	0.08	0.08	0.00
5	3317	0.12	0.24	2.53	7.15	57.56	13.42	9.04	4.13	2.95	1.57	0.45	0.18	0.45	0.21
6	5145	0.04	0.06	0.52	1.92	9.23	56.92	16.56	7.11	3.58	2.37	0.49	0.27	0.58	0.35
7	9262	0.02	0.03	0.35	0.81	3.12	10.01	59.08	13.98	6.61	3.39	0.97	0.51	0.64	0.49
8	12211	0.01	0.03	0.19	0.42	1.41	3.25	12.30	54.88	15.49	7.36	2.30	0.78	1.01	0.57
9	14343	0.01	0.03	0.04	0.25	0.85	1.66	6.14	15.40	51.14	15.68	4.99	1.46	1.38	0.97
10	13687	0.00	0.01	0.02	0.09	0.39	0.97	3.65	7.35	16.05	52.77	10.16	3.84	2.48	2.21
11	6788	0.00	0.00	0.04	0.03	0.31	0.63	2.52	4.18	9.55	20.83	43.64	8.13	6.00	4.14
12	3937	0.00	0.00	0.05	0.05	0.20	0.36	1.80	3.51	5.92	15.77	16.31	39.01	11.53	5.49
13	5049	0.00	0.02	0.08	0.20	0.26	0.59	1.64	2.81	3.90	5.82	6.75	8.69	60.75	8.48
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100.00

Table 26: The original adjusted transition matrix for year 2011

2011	ISSUER	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	28	66.67	3.92	3.92	6.87	3.92	3.92	0.00	6.86	3.92	0.00	0.00	0.00	0.00	0.00
2	172	0.58	37.80	20.36	14.53	14.53	5.81	1.16	1.16	1.16	1.74	0.00	0.58	0.58	0.00
3	740	0.41	4.46	34.32	20.41	13.92	11.22	4.46	4.73	3.11	1.35	0.27	0.68	0.54	0.14
4	1355	0.22	2.73	5.02	33.87	17.86	16.90	9.89	5.39	3.39	2.80	0.74	0.22	0.59	0.37
5	3358	0.03	0.42	1.55	4.41	30.79	18.25	14.18	11.08	9.68	6.05	1.73	0.60	0.86	0.39
6	5313	0.09	0.13	0.87	2.92	8.41	35.01	17.47	12.95	11.37	6.04	2.30	0.96	1.00	0.49
7	10005	0.01	0.05	0.34	0.73	3.27	10.63	32.41	17.27	15.51	11.63	4.41	1.64	1.33	0.76
8	12624	0.00	0.02	0.40	0.42	1.58	5.85	12.24	33.13	19.73	16.06	5.93	2.03	1.75	0.86
9	13915	0.01	0.01	0.07	0.23	0.84	2.54	7.22	14.13	35.34	22.92	9.64	3.35	2.57	1.14
10	13836	0.00	0.01	0.04	0.12	0.43	1.27	3.84	7.60	15.92	41.10	16.11	6.90	3.79	2.86
11	6588	0.00	0.00	0.02	0.12	0.43	0.59	2.34	4.08	9.59	23.35	34.35	12.80	7.62	4.72
12	3475	0.00	0.00	0.12	0.14	0.26	0.60	1.61	4.29	6.47	16.60	21.18	29.13	11.91	7.68
13	4829	0.00	0.02	0.17	0.14	0.50	0.95	2.32	3.42	4.80	9.71	12.47	12.82	43.38	9.30
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100.00

Table 27: The original adjusted transition matrix for year 2012

2012	ISSUER	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	17	52.48	5.94	17.82	11.88	5.94	0.00	5.94	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	160	0.63	42.50	17.50	14.38	12.50	4.38	1.25	3.75	1.25	0.63	0.00	0.00	1.25	0.00
3	485	1.24	3.71	36.91	23.51	8.87	7.42	6.80	6.19	1.65	2.68	0.62	0.41	0.00	0.00
4	1069	0.19	0.94	8.14	42.19	16.74	14.87	7.48	3.65	2.71	2.34	0.37	0.09	0.28	0.00
5	2390	0.08	0.17	1.55	7.91	30.21	19.12	16.40	8.20	8.12	4.85	1.59	0.71	0.84	0.25
6	5124	0.02	0.10	0.66	2.38	8.86	37.18	20.14	12.74	9.78	5.50	1.64	0.47	0.35	0.18
7	8002	0.02	0.02	0.20	0.64	3.24	12.43	33.78	18.47	15.02	10.98	2.72	1.29	0.77	0.41
8	10484	0.02	0.02	0.05	0.22	0.88	5.53	13.82	32.51	22.53	16.63	4.73	1.60	1.05	0.42
9	13227	0.00	0.01	0.02	0.13	0.42	2.07	6.32	14.54	37.65	25.73	8.13	2.83	1.21	0.95
10	15630	0.00	0.03	0.04	0.06	0.19	0.81	2.44	5.84	17.72	46.05	16.86	5.75	2.44	1.76
11	8732	0.00	0.01	0.05	0.05	0.10	0.42	0.90	2.24	7.27	25.49	39.50	14.03	5.38	4.55
12	4391	0.00	0.00	0.05	0.14	0.07	0.50	0.87	1.43	3.73	15.35	23.43	35.35	10.73	8.36
13	3737	0.05	0.00	0.11	0.21	0.19	0.51	1.36	1.93	3.21	9.34	11.72	14.40	43.06	13.91
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100.00

Table 28: The original adjusted transition matrix for year 2013

2013	ISSUER	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	14	64.65	0.00	7.07	14.14	0.00	7.07	0.00	7.07	0.00	0.00	0.00	0.00	0.00	0.00
2	105	0.00	65.72	12.38	9.52	3.81	5.71	1.90	0.95	0.00	0.00	0.00	0.00	0.00	0.00
3	380	0.00	4.21	59.47	22.63	5.26	5.26	1.58	0.53	0.53	0.53	0.00	0.00	0.00	0.00
4	939	0.11	1.28	11.82	50.05	14.27	11.71	4.58	2.24	1.49	1.60	0.32	0.21	0.00	0.32
5	1891	0.00	0.16	2.17	10.58	47.54	23.37	8.57	3.12	2.06	1.32	0.74	0.05	0.32	0.00
6	4655	0.00	0.02	0.41	2.73	8.29	49.82	20.88	9.26	4.79	2.58	0.45	0.30	0.34	0.13
7	7129	0.00	0.01	0.14	0.65	2.83	16.02	45.38	18.57	9.64	5.08	0.79	0.32	0.35	0.22
8	9255	0.02	0.01	0.05	0.70	0.91	5.52	17.33	40.86	21.03	9.95	2.12	0.64	0.42	0.43
9	13370	0.02	0.01	0.09	0.13	0.56	2.65	6.94	16.84	43.46	21.85	4.97	1.26	0.66	0.58
10	17717	0.01	0.00	0.01	0.09	0.25	0.80	2.18	6.15	19.00	52.31	12.68	3.72	1.52	1.28
11	9774	0.00	0.02	0.02	0.06	0.11	0.33	0.85	2.48	7.27	28.76	41.87	10.93	3.87	3.44
12	5104	0.00	0.02	0.00	0.04	0.12	0.24	0.45	1.31	3.51	17.05	24.28	34.54	10.40	8.05
13	3256	0.00	0.00	0.06	0.03	0.21	0.71	0.86	1.63	2.89	7.56	11.00	14.22	45.95	14.90
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100.00

Table 29: The original adjusted transition matrix for year 2014

2014	ISSUER	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	10	70.01	0.00	10.00	0.00	0.00	0.00	0.00	0.00	0.00	20.00	0.00	0.00	0.00	0.00
2	105	2.86	64.76	15.24	5.71	6.67	0.95	2.86	0.00	0.00	0.95	0.00	0.00	0.00	0.00
3	410	0.49	4.88	54.39	25.37	8.05	3.90	0.73	0.49	0.49	0.73	0.49	0.00	0.00	0.00
4	1005	0.50	0.70	9.15	58.21	15.92	9.35	1.99	1.59	1.19	1.00	0.20	0.10	0.00	0.10
5	1891	0.05	0.11	1.64	8.51	57.06	18.14	7.51	2.86	2.33	1.06	0.11	0.16	0.32	0.16
6	5131	0.00	0.00	0.68	1.68	9.14	48.43	22.33	9.71	4.31	2.81	0.39	0.18	0.21	0.14
7	7648	0.01	0.03	0.18	0.42	2.41	14.78	46.21	19.86	9.54	4.96	0.90	0.24	0.29	0.18
8	9595	0.04	0.00	0.09	0.17	0.94	4.68	16.53	42.67	21.92	9.70	2.30	0.44	0.32	0.20
9	13719	0.02	0.01	0.02	0.11	0.36	1.68	6.07	16.91	45.94	22.01	4.42	1.36	0.63	0.47
10	18489	0.00	0.01	0.02	0.05	0.17	0.55	1.87	5.68	18.12	52.69	14.32	3.76	1.59	1.17
11	9212	0.01	0.03	0.01	0.04	0.08	0.23	0.76	2.03	6.16	26.45	45.93	11.44	3.72	3.10
12	4375	0.00	0.00	0.02	0.00	0.02	0.07	0.53	0.98	3.68	14.19	25.39	39.98	9.42	5.71
13	2985	0.00	0.00	0.07	0.10	0.07	0.30	0.80	1.34	2.18	7.10	11.22	12.63	49.21	14.97
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100.00

Appendix B.2

The aggregated adjusted migration probabilities for the period of 2007-2014 are reported in Table 30-37.

Table 30: The aggregated adjusted transition matrix for year 2007

2007	Issuers	A+	A	B+	B	C	D	F
A+	91	53.54	13.13	13.13	10.10	9.09	1.01	0.00
A	322	2.80	27.33	24.53	11.80	20.81	12.42	0.31
B+	1132	0.88	2.39	28.62	11.93	39.13	16.97	0.08
B	3181	0.03	0.16	2.77	23.45	50.09	23.19	0.31
C	18287	0.03	0.22	0.91	1.96	52.82	43.40	0.66
D	38163	0.03	0.03	0.13	0.34	11.81	84.68	2.99
F	0	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Table 31: The aggregated adjusted transition matrix for year 2008

2008	Issuers	A+	A	B+	B	C	D	F
A+	63	79.00	3.00	2.00	0.00	16.00	0.00	0.00
A	155	2.58	53.55	12.26	9.68	14.20	7.73	0.00
B+	727	0.28	2.61	53.37	18.16	17.61	7.56	0.41
B	1547	0.06	1.16	5.17	47.32	33.87	11.64	0.78
C	18728	0.03	0.07	0.54	1.68	69.20	27.38	1.11
D	50033	0.02	0.07	0.07	0.22	7.74	88.28	3.61
F	0	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Table 32: The aggregated adjusted transition matrix for year 2009

2009	Issuers	A+	A	B+	B	C	D	F
A+	35	68.32	0.00	10.89	2.97	8.91	8.91	0.00
A	130	0.77	64.62	16.91	8.46	9.24	0.00	0.00
B+	622	0.48	6.59	61.26	8.84	16.07	6.59	0.16
B	1340	0.00	0.52	10.82	52.40	28.88	7.23	0.15
C	18243	0.01	0.11	0.69	2.59	68.79	27.35	0.47
D	53380	0.01	0.10	0.08	0.19	7.38	89.02	3.22
F	0	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Table 33: The aggregated adjusted transition matrix for year 2010

2010	Issuers	A+	A	B+	B	C	D	F
A+	29	81.44	3.09	3.09	0.00	9.28	3.09	0.00
A	230	0.00	85.23	6.52	3.91	3.04	1.30	0.00
B+	706	0.28	4.39	62.75	15.16	13.88	3.40	0.14
B	1329	0.08	0.68	8.13	57.49	28.73	4.89	0.00
C	17724	0.04	0.07	0.81	2.32	76.72	19.64	0.40
D	56015	0.00	0.02	0.07	0.20	7.95	89.19	2.56
F	0	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Table 34: The aggregated adjusted transition matrix for year 2011

2011	Issuers	A+	A	B+	B	C	D	F
A+	28	66.67	3.92	3.92	6.87	7.84	10.78	0.00
A	172	0.58	37.80	20.36	14.53	21.51	5.22	0.00
B+	740	0.41	4.46	34.32	20.41	29.60	10.68	0.12
B	1355	0.22	2.73	5.02	33.87	44.65	13.13	0.37
C	18676	0.04	0.14	0.71	2.01	53.50	42.99	0.61
D	55267	0.00	0.01	0.14	0.22	9.43	87.13	3.06
F	0	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Table 35: The aggregated adjusted transition matrix for year 2012

2012	Issuers	A+	A	B+	B	C	D	F
A+	17	52.48	5.94	17.82	11.88	11.88	0.00	0.00
A	160	0.63	42.50	17.50	14.38	18.13	6.86	0.00
B+	485	1.24	3.71	36.91	23.51	23.08	11.55	0.00
B	1069	0.19	0.94	8.14	42.19	39.09	9.44	0.00
C	15516	0.03	0.07	0.56	2.33	57.48	39.21	0.31
D	56201	0.01	0.02	0.04	0.12	7.27	89.47	3.08
F	0	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Table 36: The aggregated adjusted transition matrix for year 2013

2013	Issuers	A+	A	B+	B	C	D	F
A+	14	64.65	0.00	7.07	14.14	7.07	7.07	0.00
A	105	0.00	65.72	12.38	9.52	11.42	0.95	0.00
B+	380	0.00	4.21	59.47	22.63	12.10	1.59	0.00
B	939	0.11	1.28	11.82	50.05	30.56	5.86	0.32
C	13675	0.00	0.03	0.51	2.73	71.36	25.20	0.16
D	58476	0.01	0.01	0.04	0.17	7.45	89.63	2.70
F	0	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Table 37: The aggregated adjusted transition matrix for year 2014

2014	Issuers	A+	A	B+	B	C	D	F
A+	10	70.01	0.00	10.00	0.00	0.00	19.98	0.00
A	105	2.86	64.76	15.24	5.71	10.48	0.95	0.00
B+	410	0.49	4.88	54.39	25.37	12.68	2.19	0.00
B	1005	0.50	0.70	9.15	58.21	27.26	4.08	0.10
C	14670	0.01	0.03	0.54	1.90	71.66	25.70	0.15
D	58375	0.01	0.01	0.03	0.08	6.64	91.03	2.20
F	0	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Appendix C

Statistic description of the macroeconomic variables are shown in Table 38.

Table 38: Descriptive statistics macroeconomic variables

Variables		Mean	StdDev	Min	Max
National accounts	GDP	0.724	1.688	-2.873	2.424
	OUTG	-0.232	1.396	-2.137	2.124
	UNEM	8.844	1.079	7.063	10.291
	EMP	64.121	0.378	63.650	64.925
	IPROD	102.277	5.818	95.517	112.560
	CPI	101.871	3.593	96.376	106.245
	PPI	100.102	3.199	95.425	103.608
Financial markets	SINTS	1.480	1.712	0.080	4.848
	LINTS	3.128	0.945	1.666	4.303
	SPL-S	1.460	1.062	-0.399	2.420
	EXCH	100.236	3.366	96.010	104.244
Stock makets	CAC40	-0.837	21.082	-42	22.300
	VCAC4	23.925	7.399	15.139	37.938
Composite index	CLI	98.748	3.291	91.022	101.562

Appendix D

The estimated transition probabilities obtained by the two-step estimation method over the period 2007 to 2014 are given in Tables 39-46.

Table 39: Estimated transition matrix for year 2007

2007	A+	A	B+	B	C	D	F
A+	0.74753	0.036174	0.030464	0.025777	0.039879	0.052946	0.067227
A	0.11155	0.21599	0.27629	0.2181	0.16016	0.017909	1.29E-05
B+	0.022129	0.10565	0.24211	0.29118	0.29874	0.040178	1.74E-05
B	2.91E-05	0.002426	0.04194	0.20708	0.6276	0.12092	3.89E-06
C	6.73E-09	6.08E-06	0.000749	0.01909	0.45344	0.52648	0.000231
D	1.14E-10	1.15E-07	2.07E-05	0.000945	0.092334	0.87609	0.030611
F	0	0	0	0	0	0	1

Table 40: Estimated transition matrix for year 2008

2008	A+	A	B+	B	C	D	F
A+	0.78177	0.033161	0.027576	0.023062	0.035135	0.045404	0.053897
A	0.13664	0.23636	0.27728	0.20184	0.13474	0.013151	7.43E-06
B+	0.025834	0.1162	0.25293	0.28983	0.28039	0.034812	1.30E-05
B	0.001321	0.034671	0.20966	0.38882	0.35104	0.014483	2.07E-08
C	2.74E-08	1.82E-05	0.001687	0.033274	0.53558	0.42936	8.94E-05
D	1.06E-10	1.08E-07	1.97E-05	0.00091	0.090539	0.87715	0.031384
F	0	0	0	0	0	0	1

Table 41: Estimated transition matrix for year 2009

2009	A+	A	B+	B	C	D	F
A+	0.79228	0.032151	0.026628	0.022185	0.033629	0.04308	0.050051
A	0.14555	0.24262	0.27674	0.19621	0.12701	0.011862	6.19E-06
B+	0.027143	0.11975	0.25631	0.28913	0.27445	0.033202	1.19E-05
B	0.003708	0.067052	0.28898	0.38972	0.24455	0.005985	3.05E-09
C	4.26E-08	2.56E-05	0.00217	0.039379	0.55994	0.39842	6.50E-05
D	1.04E-10	1.06E-07	1.94E-05	0.000899	0.089962	0.87748	0.031638
F	0	0	0	0	0	0	1

Table 42: Estimated transition matrix for year 2010

2010	A+	A	B+	B	C	D	F
A+	0.80127	0.031255	0.025792	0.021417	0.032323	0.041089	0.046849
A	0.15372	0.24796	0.27591	0.19114	0.12043	0.01082	5.27E-06
B+	0.028341	0.12291	0.25924	0.28842	0.26924	0.031836	1.09E-05
B	0.00847	0.11042	0.35339	0.3598	0.16539	0.002534	5.19E-10
C	6.26E-08	3.44E-05	0.002695	0.045452	0.58006	0.37171	4.88E-05
D	1.01E-10	1.04E-07	1.91E-05	0.00089	0.089456	0.87777	0.031863
F	0	0	0	0	0	0	1

Table 43: Estimated transition matrix for year 2011

2011	A+	A	B+	B	C	D	F
A+	0.765	0.034688	0.029029	0.024419	0.037489	0.049104	0.060272
A	0.12365	0.22635	0.27728	0.21019	0.14714	0.015372	9.80E-06
B+	0.023922	0.11086	0.24759	0.29065	0.28955	0.037416	1.51E-05
B	0.000221	0.010337	0.10551	0.31922	0.51797	0.046743	3.21E-07
C	1.37E-08	1.06E-05	0.001132	0.025381	0.49558	0.47776	0.000145
D	1.10E-10	1.11E-07	2.02E-05	0.000927	0.091433	0.87662	0.030995
F	0	0	0	0	0	0	1

Table 44: Estimated transition matrix for year 2012

2012	A+	A	B+	B	C	D	F
A+	0.77548	0.033744	0.028129	0.023577	0.036023	0.046791	0.05625
A	0.1316	0.23261	0.2774	0.20506	0.13937	0.013959	8.26E-06
B+	0.025094	0.11416	0.25092	0.29018	0.28386	0.035782	1.38E-05
B	0.00069	0.022523	0.16608	0.3709	0.41661	0.023203	6.04E-08
C	2.11E-08	1.48E-05	0.001452	0.030074	0.52071	0.44764	0.000108
D	1.07E-10	1.09E-07	1.99E-05	0.000917	0.090878	0.87695	0.031236
F	0	0	0	0	0	0	1

Table 45: Estimated transition matrix for year 2013

2013	A+	A	B+	B	C	D	F
A+	0.79685	0.0317	0.026206	0.021797	0.032968	0.042068	0.048414
A	0.14964	0.24534	0.27636	0.19366	0.12366	0.011326	5.71E-06
B+	0.027742	0.12134	0.2578	0.28878	0.27182	0.032507	1.14E-05
B	0.00568	0.087076	0.32292	0.37793	0.20248	0.003916	1.26E-09
C	5.18E-08	2.97E-05	0.002422	0.042359	0.57026	0.38487	5.63E-05
D	1.03E-10	1.05E-07	1.93E-05	0.000895	0.089706	0.87763	0.031752
F	0	0	0	0	0	0	1

Table 46: Estimated transition matrix for year 2014

2014	A+	A	B+	B	C	D	F
A+	0.79707	0.031677	0.026185	0.021777	0.032935	0.042018	0.048333
A	0.14984	0.24547	0.27634	0.19354	0.1235	0.0113	5.69E-06
B+	0.027772	0.12142	0.25787	0.28876	0.27169	0.032472	1.14E-05
B	0.005799	0.088172	0.32455	0.37716	0.20049	0.003832	1.21E-09
C	5.23E-08	2.99E-05	0.002435	0.042512	0.57077	0.3842	5.59E-05
D	1.02E-10	1.05E-07	1.92E-05	0.000894	0.089693	0.87764	0.031757
F	0	0	0	0	0	0	1

Appendix E

The stressed transition matrices under positive and negative shock to the factor and under "mutatis mutandis" scenario over the period 2008 to 2014 are reported in Appendix E.1 and E.2 respectively. Appendix E.3 and E.4 report the above results over the period 2010 to 2014 respectively.

The stressed transition matrices under positive and negative shock to the factor and under "ceteris paribus" scenario over the period 2008 to 2014 are reported in Appendix E.5 and E.6 respectively. Appendix E.7 and E.8 report the above results over the period 2010 to 2014 respectively.

Appendix E.1

The stressed transition matrices under positive shock to the factor over the period 2008 to 2014, under "mutatis mutandis" scenario are reported in Table 47-53.

Table 47: Stressed transition matrix for year 2008-Positive shock

2008	A+	A	B+	B	C	D	F
A+	0.79975	0.031409	0.025935	0.021548	0.032546	0.041427	0.047387
A	0.1523	0.24706	0.27608	0.19202	0.12155	0.010993	5.42E-06
B+	0.028132	0.12236	0.25874	0.28855	0.27014	0.032067	1.11E-05
B	0.007391	0.10191	0.34324	0.36676	0.17774	0.002953	7.08E-10
C	5.86E-08	3.27E-05	0.002597	0.044361	0.57671	0.37625	5.13E-05
D	1.02E-10	1.05E-07	1.92E-05	0.000891	0.089543	0.87772	0.031825
F	0	0	0	0	0	0	1

Table 48: Stressed transition matrix for year 2009-Positive shock

2009	A+	A	B+	B	C	D	F
A+	0.79937	0.031448	0.025971	0.021581	0.032601	0.041511	0.047521
A	0.15195	0.24683	0.27612	0.19224	0.12182	0.011036	5.46E-06
B+	0.028081	0.12223	0.25862	0.28858	0.27036	0.032125	1.11E-05
B	0.007142	0.099871	0.34065	0.36838	0.18089	0.003066	7.64E-10
C	5.77E-08	3.23E-05	0.002573	0.044094	0.57587	0.37738	5.19E-05
D	1.02E-10	1.05E-07	1.92E-05	0.000892	0.089564	0.87771	0.031815
F	0	0	0	0	0	0	1

Table 49: Stressed transition matrix for year 2010-Positive shock

2010	A+	A	B+	B	C	D	F
A+	0.80405	0.030972	0.025529	0.021177	0.031916	0.040474	0.045878
A	0.15635	0.24961	0.27559	0.18953	0.11841	0.010509	5.01E-06
B+	0.028726	0.12391	0.26015	0.28817	0.26761	0.031417	1.06E-05
B	0.010811	0.12716	0.37061	0.34536	0.14416	0.001904	2.93E-10
C	7.06E-08	3.77E-05	0.002882	0.047503	0.58611	0.36343	4.46E-05
D	1.01E-10	1.04E-07	1.90E-05	0.000887	0.089297	0.87786	0.031935
F	0	0	0	0	0	0	1

Table 50: Stressed transition matrix for year 2011-Positive shock

2011	A+	A	B+	B	C	D	F
A+	0.76623	0.034579	0.028925	0.024322	0.037319	0.048834	0.059795
A	0.12455	0.22708	0.27731	0.20961	0.14623	0.015202	9.61E-06
B+	0.024055	0.11124	0.24797	0.29061	0.28889	0.037224	1.49E-05
B	0.000254	0.011366	0.11172	0.32632	0.50705	0.043292	2.66E-07
C	1.44E-08	1.10E-05	0.001165	0.025892	0.49853	0.47426	0.00014
D	1.10E-10	1.11E-07	2.01E-05	0.000926	0.091369	0.87666	0.031023
F	0	0	0	0	0	0	1

Table 51: Stressed transition matrix for year 2012-Positive shock

2012	A+	A	B+	B	C	D	F
A+	0.77596	0.0337	0.028087	0.023538	0.035956	0.046685	0.056069
A	0.13198	0.23289	0.27739	0.20481	0.13902	0.013897	8.19E-06
B+	0.025149	0.11431	0.25107	0.29015	0.2836	0.035708	1.37E-05
B	0.000725	0.023299	0.16924	0.37267	0.41166	0.022415	5.57E-08
C	2.15E-08	1.50E-05	0.001469	0.030307	0.52185	0.44625	0.000106
D	1.07E-10	1.09E-07	1.99E-05	0.000916	0.090852	0.87696	0.031247
F	0	0	0	0	0	0	1

Table 52: Stressed transition matrix for year 2013-Positive shock

2013	A+	A	B+	B	C	D	F
A+	0.79703	0.031682	0.026189	0.021781	0.032941	0.042028	0.048349
A	0.1498	0.24545	0.27634	0.19356	0.12353	0.011305	5.69E-06
B+	0.027766	0.1214	0.25786	0.28877	0.27172	0.032479	1.14E-05
B	0.005775	0.087953	0.32422	0.37731	0.20089	0.003849	1.22E-09
C	5.22E-08	2.99E-05	0.002433	0.042482	0.57067	0.38433	5.59E-05
D	1.02E-10	1.05E-07	1.92E-05	0.000894	0.089696	0.87763	0.031756
F	0	0	0	0	0	0	1

Table 53: Stressed transition matrix for year 2014-Positive shock

2014	A+	A	B+	B	C	D	F
A+	0.79715	0.03167	0.026178	0.021771	0.032924	0.042002	0.048307
A	0.14991	0.24552	0.27633	0.1935	0.12345	0.011291	5.68E-06
B+	0.027782	0.12144	0.2579	0.28876	0.27165	0.032461	1.13E-05
B	0.005838	0.088525	0.32507	0.37691	0.19986	0.003806	1.19E-09
C	5.24E-08	3.00E-05	0.002439	0.042561	0.57093	0.38398	5.57E-05
D	1.02E-10	1.05E-07	1.92E-05	0.000894	0.089689	0.87764	0.031759
F	0	0	0	0	0	0	1

Appendix E.2

The stressed transition matrices under negative shock to the factor over the period 2008 to 2014, under "mutatis mutandis" scenario are reported in Table 54-60.

Table 54: Stressed transition matrix for year 2008-Negative shock

2008	A+	A	B+	B	C	D	F
A+	0.76289	0.034873	0.029207	0.024586	0.037781	0.049569	0.061096
A	0.12211	0.22509	0.27721	0.21119	0.14871	0.015666	1.01E-05
B+	0.023695	0.11021	0.24692	0.29074	0.29068	0.037747	1.53E-05
B	0.000175	0.008763	0.095403	0.30655	0.53595	0.053157	4.42E-07
C	1.25E-08	9.90E-06	0.001077	0.024526	0.49049	0.48374	0.000153
D	1.10E-10	1.12E-07	2.02E-05	0.000929	0.091544	0.87656	0.030948
F	0	0	0	0	0	0	1

Table 55: Stressed transition matrix for year 2009-Negative shock

2009	A+	A	B+	B	C	D	F
A+	0.78504	0.032851	0.027284	0.022791	0.034668	0.04468	0.052687
A	0.13935	0.23831	0.27715	0.20012	0.13232	0.012742	7.02E-06
B+	0.026232	0.11729	0.25398	0.28963	0.27856	0.034309	1.27E-05
B	0.001835	0.042942	0.23391	0.39323	0.31694	0.011144	1.16E-08
C	3.14E-08	2.02E-05	0.001824	0.03507	0.54324	0.41976	8.11E-05
D	1.05E-10	1.08E-07	1.96E-05	0.000907	0.090361	0.87725	0.031462
F	0	0	0	0	0	0	1

Table 56: Stressed transition matrix for year 2010-Negative shock

2010	A+	A	B+	B	C	D	F
A+	0.79847	0.031537	0.026055	0.021658	0.032732	0.041709	0.047837
A	0.15112	0.2463	0.27621	0.19275	0.12248	0.011139	5.55E-06
B+	0.02796	0.12191	0.25833	0.28865	0.27088	0.032261	1.12E-05
B	0.006587	0.095175	0.33445	0.37201	0.18842	0.003348	9.14E-10
C	5.55E-08	3.13E-05	0.002519	0.043469	0.57388	0.38004	5.34E-05
D	1.02E-10	1.05E-07	1.92E-05	0.000893	0.089615	0.87768	0.031792
F	0	0	0	0	0	0	1

Table 57: Stressed transition matrix for year 2011-Negative shock

2011	A+	A	B+	B	C	D	F
A+	0.76377	0.034796	0.029133	0.024517	0.03766	0.049376	0.060752
A	0.12275	0.22562	0.27724	0.21078	0.14806	0.015543	9.99E-06
B+	0.023789	0.11048	0.2472	0.2907	0.29021	0.037609	1.52E-05
B	0.000193	0.00939	0.099522	0.31189	0.52859	0.050412	3.87E-07
C	1.30E-08	1.02E-05	0.001099	0.024878	0.49261	0.48126	0.00015
D	1.10E-10	1.12E-07	2.02E-05	0.000929	0.091498	0.87659	0.030968
F	0	0	0	0	0	0	1

Table 58: Stressed transition matrix for year 2012-Negative shock

2012	A+	A	B+	B	C	D	F
A+	0.775	0.033788	0.028171	0.023616	0.036091	0.046897	0.056432
A	0.13123	0.23232	0.2774	0.2053	0.13973	0.014022	8.32E-06
B+	0.025038	0.114	0.25076	0.2902	0.28413	0.035857	1.38E-05
B	0.000656	0.021769	0.16295	0.36906	0.42155	0.024015	6.54E-08
C	2.07E-08	1.46E-05	0.001436	0.029842	0.51957	0.44903	0.000109
D	1.08E-10	1.10E-07	1.99E-05	0.000917	0.090903	0.87693	0.031225
F	0	0	0	0	0	0	1

Table 59: Stressed transition matrix for year 2013-Negative shock

2013	A+	A	B+	B	C	D	F
A+	0.79667	0.031718	0.026223	0.021812	0.032994	0.042109	0.048478
A	0.14948	0.24523	0.27638	0.19377	0.1238	0.011347	5.73E-06
B+	0.027718	0.12127	0.25774	0.2888	0.27193	0.032534	1.14E-05
B	0.005586	0.086206	0.32161	0.37853	0.20408	0.003984	1.31E-09
C	5.14E-08	2.95E-05	0.002411	0.042237	0.56986	0.38541	5.66E-05
D	1.03E-10	1.05E-07	1.93E-05	0.000895	0.089716	0.87762	0.031747
F	0	0	0	0	0	0	1

Table 60: Stressed transition matrix for year 2014-Negative shock

2014	A+	A	B+	B	C	D	F
A+	0.797	0.031684	0.026192	0.021784	0.032945	0.042034	0.048359
A	0.14978	0.24543	0.27635	0.19358	0.12355	0.011308	5.69E-06
B+	0.027763	0.12139	0.25785	0.28877	0.27173	0.032483	1.14E-05
B	0.005761	0.08782	0.32403	0.3774	0.20113	0.003859	1.22E-09
C	5.21E-08	2.99E-05	0.002431	0.042463	0.57061	0.38441	5.60E-05
D	1.02E-10	1.05E-07	1.92E-05	0.000894	0.089698	0.87763	0.031756
F	0	0	0	0	0	0	1

Appendix E.3

The stressed transition matrices under positive shock to the factor over the period 2010 to 2014, under "mutatis mutandis" scenario are reported in Table 61-65.

Table 61: Stressed transition matrix for year 2010-Positive shock

2010	A+	A	B+	B	C	D	F
A+	0.81825	0.029479	0.024154	0.019926	0.029815	0.037333	0.041039
A	0.17063	0.25792	0.27336	0.18094	0.10814	0.009002	3.83E-06
B+	0.030822	0.12925	0.26483	0.28674	0.25907	0.029288	9.25E-06
B	0.034455	0.23596	0.42085	0.2459	0.062462	0.000374	1.26E-11
C	1.31E-07	6.07E-05	0.004069	0.059457	0.61531	0.32107	2.74E-05
D	9.74E-11	1.01E-07	1.86E-05	0.000871	0.088468	0.87833	0.03231
F	0	0	0	0	0	0	1

Table 62: Stressed transition matrix for year 2011-Positive shock

2011	A+	A	B+	B	C	D	F
A+	0.77262	0.034006	0.028378	0.023809	0.036427	0.047424	0.057339
A	0.12937	0.23089	0.2774	0.20649	0.14149	0.014338	8.66E-06
B+	0.024765	0.11324	0.25	0.29032	0.28543	0.036227	1.41E-05
B	0.000509	0.018332	0.14786	0.35908	0.44583	0.028398	9.67E-08
C	1.87E-08	1.35E-05	0.001357	0.028713	0.51387	0.45593	0.000117
D	1.08E-10	1.10E-07	1.99E-05	0.00092	0.091031	0.87686	0.031169
F	0	0	0	0	0	0	1

Table 63: Stressed transition matrix for year 2012-Positive shock

2012	A+	A	B+	B	C	D	F
A+	0.77847	0.033469	0.027868	0.023333	0.035602	0.046131	0.055126
A	0.13397	0.23439	0.27736	0.20354	0.13717	0.013572	7.85E-06
B+	0.025442	0.11512	0.25187	0.29002	0.28222	0.03532	1.34E-05
B	0.000942	0.027744	0.18623	0.38088	0.38556	0.018639	3.65E-08
C	2.39E-08	1.63E-05	0.00156	0.031557	0.5278	0.43897	9.86E-05
D	1.07E-10	1.09E-07	1.98E-05	0.000914	0.090717	0.87704	0.031306
F	0	0	0	0	0	0	1

Table 64: Stressed transition matrix for year 2013-Positive shock

2013	A+	A	B+	B	C	D	F
A+	0.79798	0.031587	0.026101	0.0217	0.032804	0.041819	0.048013
A	0.15067	0.24601	0.27625	0.19303	0.12284	0.011196	5.60E-06
B+	0.027893	0.12174	0.25817	0.28869	0.27117	0.032335	1.13E-05
B	0.006298	0.092653	0.33098	0.37391	0.19265	0.003513	1.01E-09
C	5.43E-08	3.08E-05	0.002489	0.043129	0.57278	0.38151	5.43E-05
D	1.02E-10	1.05E-07	1.92E-05	0.000893	0.089643	0.87766	0.03178
F	0	0	0	0	0	0	1

Table 65: Stressed transition matrix for year 2014-Positive shock

2014	A+	A	B+	B	C	D	F
A+	0.79753	0.031632	0.026143	0.021739	0.032869	0.041918	0.048173
A	0.15026	0.24574	0.2763	0.19328	0.12317	0.011247	5.64E-06
B+	0.027833	0.12158	0.25802	0.28873	0.27143	0.032404	1.13E-05
B	0.006044	0.090393	0.32778	0.37556	0.19655	0.00367	1.10E-09
C	5.33E-08	3.04E-05	0.002462	0.042819	0.57178	0.38286	5.51E-05
D	1.02E-10	1.05E-07	1.92E-05	0.000894	0.089668	0.87765	0.031769
F	0	0	0	0	0	0	1

Appendix E.4

The stressed transition matrices under negative shock to the factor over the period 2010 to 2014, under "mutatis mutandis" scenario are reported in Table 66-70.

Table 66: Stressed transition matrix for year 2010-Negative shock

2010	A+	A	B+	B	C	D	F
A+	0.78337	0.033009	0.027433	0.02293	0.034906	0.045048	0.053301
A	0.13796	0.23731	0.27722	0.201	0.13355	0.012949	7.23E-06
B+	0.026028	0.11673	0.25344	0.28973	0.27949	0.034565	1.29E-05
B	0.001553	0.038547	0.22147	0.39143	0.33424	0.012753	1.56E-08
C	2.93E-08	1.91E-05	0.001753	0.034145	0.53935	0.42465	8.53E-05
D	1.06E-10	1.08E-07	1.96E-05	0.000909	0.090452	0.8772	0.031422
F	0	0	0	0	0	0	1

Table 67: Stressed transition matrix for year 2011-Negative shock

2011	A+	A	B+	B	C	D	F
A+	0.75724	0.035361	0.029676	0.025029	0.038559	0.050813	0.063322
A	0.11811	0.22174	0.27696	0.2138	0.15291	0.01647	1.11E-05
B+	0.023102	0.1085	0.24514	0.29093	0.29367	0.038636	1.61E-05
B	9.17E-05	0.005555	0.071796	0.27063	0.57836	0.073571	1.02E-06
C	9.97E-09	8.27E-06	0.000942	0.022373	0.47686	0.49964	0.000179
D	1.12E-10	1.13E-07	2.04E-05	0.000935	0.091837	0.87639	0.030822
F	0	0	0	0	0	0	1

Table 68: Stressed transition matrix for year 2012-Negative shock

2012	A+	A	B+	B	C	D	F
A+	0.77248	0.034019	0.02839	0.023821	0.036446	0.047455	0.057393
A	0.12926	0.23081	0.2774	0.20656	0.1416	0.014356	8.68E-06
B+	0.024749	0.1132	0.24996	0.29033	0.28551	0.036249	1.41E-05
B	0.000501	0.018145	0.14699	0.35845	0.44724	0.028675	9.89E-08
C	1.86E-08	1.35E-05	0.001352	0.028648	0.51353	0.45634	0.000117
D	1.08E-10	1.10E-07	2.00E-05	0.00092	0.091038	0.87686	0.031166
F	0	0	0	0	0	0	1

Table 69: Stressed transition matrix for year 2013-Negative shock

2013	A+	A	B+	B	C	D	F
A+	0.79571	0.031812	0.026311	0.021893	0.033132	0.042319	0.048817
A	0.14861	0.24467	0.27646	0.1943	0.12449	0.011457	5.83E-06
B+	0.027592	0.12094	0.25743	0.28887	0.27248	0.032679	1.15E-05
B	0.005117	0.081737	0.31468	0.38153	0.21258	0.004361	1.58E-09
C	4.93E-08	2.86E-05	0.002357	0.041601	0.56772	0.38823	5.83E-05
D	1.03E-10	1.05E-07	1.93E-05	0.000896	0.08977	0.87759	0.031723
F	0	0	0	0	0	0	1

Table 70: Stressed transition matrix for year 2014-Negative shock

2014	A+	A	B+	B	C	D	F
A+	0.79662	0.031722	0.026227	0.021816	0.033001	0.042119	0.048494
A	0.14943	0.24521	0.27638	0.19379	0.12383	0.011352	5.73E-06
B+	0.027712	0.12126	0.25772	0.2888	0.27195	0.032541	1.14E-05
B	0.005563	0.085989	0.32129	0.37868	0.20448	0.004002	1.32E-09
C	5.13E-08	2.95E-05	0.002409	0.042207	0.56976	0.38554	5.67E-05
D	1.03E-10	1.05E-07	1.93E-05	0.000895	0.089719	0.87762	0.031746
F	0	0	0	0	0	0	1

Appendix E.5

The stressed transition matrices under positive shock to the factor over the period 2008 to 2014, under "ceteris paribus" scenario are reported in Table 71-77.

Table 71: Stressed transition matrix for year 2008-Positive shock

2008	A+	A	B+	B	C	D	F
A+	0.79975	0.031409	0.025935	0.021548	0.032546	0.041427	0.047387
A	0.1523	0.24706	0.27608	0.19202	0.12155	0.010993	5.42E-06
B+	0.028132	0.12236	0.25874	0.28855	0.27014	0.032067	1.11E-05
B	0.007391	0.10191	0.34324	0.36676	0.17774	0.002953	7.08E-10
C	5.86E-08	3.27E-05	0.002597	0.044361	0.57671	0.37625	5.13E-05
D	1.02E-10	1.05E-07	1.92E-05	0.000891	0.089543	0.87772	0.031825
F	0	0	0	0	0	0	1

Table 72: Stressed transition matrix for year 2009-Positive shock

2009	A+	A	B+	B	C	D	F
A+	0.78907	0.032464	0.02692	0.022455	0.034091	0.043789	0.051213
A	0.14276	0.24071	0.27695	0.19796	0.12936	0.012248	6.55E-06
B+	0.026734	0.11865	0.25528	0.28936	0.27628	0.033692	1.22E-05
B	0.002726	0.055292	0.26451	0.39358	0.27595	0.007943	5.57E-09
C	3.72E-08	2.30E-05	0.002009	0.037409	0.55258	0.4079	7.17E-05
D	1.04E-10	1.07E-07	1.95E-05	0.000903	0.09014	0.87738	0.03156
F	0	0	0	0	0	0	1

Table 73: Stressed transition matrix for year 2010-Positive shock

2010	A+	A	B+	B	C	D	F
A+	0.7847	0.032883	0.027314	0.022819	0.034716	0.044754	0.05281
A	0.13907	0.23811	0.27717	0.2003	0.13257	0.012783	7.06E-06
B+	0.026191	0.11718	0.25387	0.28965	0.27874	0.03436	1.27E-05
B	0.001775	0.042026	0.23139	0.39294	0.32041	0.011453	1.23E-08
C	3.10E-08	2.00E-05	0.00181	0.034882	0.54246	0.42075	8.19E-05
D	1.05E-10	1.08E-07	1.96E-05	0.000907	0.090379	0.87724	0.031454
F	0	0	0	0	0	0	1

Table 74: Stressed transition matrix for year 2011-Positive shock

2011	A+	A	B+	B	C	D	F
A+	0.78294	0.033049	0.027471	0.022965	0.034967	0.045143	0.05346
A	0.1376	0.23706	0.27724	0.20122	0.13387	0.013003	7.28E-06
B+	0.025976	0.11659	0.2533	0.28976	0.27973	0.034631	1.29E-05
B	0.001488	0.037478	0.2183	0.39082	0.33872	0.013197	1.68E-08
C	2.87E-08	1.89E-05	0.001735	0.03391	0.53834	0.42591	8.64E-05
D	1.06E-10	1.08E-07	1.97E-05	0.000909	0.090475	0.87718	0.031412
F	0	0	0	0	0	0	1

Table 75: Stressed transition matrix for year 2012-Positive shock

2012	A+	A	B+	B	C	D	F
A+	0.78224	0.033116	0.027534	0.023023	0.035068	0.045299	0.053722
A	0.13702	0.23664	0.27726	0.20159	0.13439	0.013092	7.37E-06
B+	0.025891	0.11636	0.25308	0.2898	0.28012	0.034739	1.30E-05
B	0.001385	0.035773	0.21311	0.38967	0.3461	0.013956	1.91E-08
C	2.79E-08	1.84E-05	0.001706	0.033528	0.53668	0.42798	8.82E-05
D	1.06E-10	1.08E-07	1.97E-05	0.00091	0.090514	0.87716	0.031395
F	0	0	0	0	0	0	1

Table 76: Stressed transition matrix for year 2013-Positive shock

2013	A+	A	B+	B	C	D	F
A+	0.78195	0.033143	0.027559	0.023047	0.035108	0.045362	0.053827
A	0.13679	0.23647	0.27727	0.20174	0.1346	0.013128	7.40E-06
B+	0.025857	0.11626	0.25299	0.28982	0.28028	0.034783	1.30E-05
B	0.001346	0.035108	0.21104	0.38917	0.34907	0.01427	2.00E-08
C	2.76E-08	1.83E-05	0.001695	0.033375	0.53602	0.4288	8.89E-05
D	1.06E-10	1.08E-07	1.97E-05	0.00091	0.090529	0.87715	0.031388
F	0	0	0	0	0	0	1

Table 77: Stressed transition matrix for year 2014-Positive shock

2014	A+	A	B+	B	C	D	F
A+	0.78184	0.033153	0.02757	0.023056	0.035124	0.045387	0.053869
A	0.1367	0.2364	0.27727	0.2018	0.13468	0.013142	7.42E-06
B+	0.025843	0.11623	0.25295	0.28982	0.28035	0.0348	1.30E-05
B	0.001331	0.034845	0.21021	0.38896	0.35025	0.014397	2.04E-08
C	2.75E-08	1.82E-05	0.00169	0.033315	0.53575	0.42913	8.92E-05
D	1.06E-10	1.08E-07	1.97E-05	0.00091	0.090535	0.87715	0.031386
F	0	0	0	0	0	0	1

Appendix E.6

The stressed transition matrices under negative shock to the factor over the period 2008 to 2014, under "ceteris paribus" scenario are reported in Table 78-84.

Table 78: Stressed transition matrix for year 2008-Negative shock

2008	A+	A	B+	B	C	D	F
A+	0.76289	0.034873	0.029207	0.024586	0.037781	0.049569	0.061096
A	0.12211	0.22509	0.27721	0.21119	0.14871	0.015666	1.01E-05
B+	0.023695	0.11021	0.24692	0.29074	0.29068	0.037747	1.53E-05
B	0.000175	0.008763	0.095403	0.30655	0.53595	0.053157	4.42E-07
C	1.25E-08	9.90E-06	0.001077	0.024526	0.49049	0.48374	0.000153
D	1.10E-10	1.12E-07	2.02E-05	0.000929	0.091544	0.87656	0.030948
F	0	0	0	0	0	0	1

Table 79: Stressed transition matrix for year 2009-Negative shock

2009	A+	A	B+	B	C	D	F
A+	0.77432	0.033851	0.02823	0.023671	0.036187	0.047048	0.056691
A	0.13069	0.23191	0.2774	0.20564	0.14023	0.014112	8.42E-06
B+	0.02496	0.11378	0.25054	0.29024	0.2845	0.035963	1.39E-05
B	0.00061	0.020731	0.15854	0.36635	0.42856	0.025211	7.32E-08
C	2.01E-08	1.43E-05	0.001413	0.029514	0.51794	0.45101	0.000111
D	1.08E-10	1.10E-07	1.99E-05	0.000918	0.09094	0.87691	0.031209
F	0	0	0	0	0	0	1

Table 80: Stressed transition matrix for year 2010-Negative shock

2010	A+	A	B+	B	C	D	F
A+	0.7788	0.033438	0.027838	0.023306	0.035555	0.046058	0.055001
A	0.13424	0.23459	0.27735	0.20337	0.13692	0.013529	7.81E-06
B+	0.025481	0.11523	0.25198	0.29	0.28203	0.035269	1.34E-05
B	0.000975	0.028386	0.18855	0.38183	0.38207	0.018178	3.45E-08
C	2.42E-08	1.65E-05	0.001572	0.031727	0.52859	0.438	9.76E-05
D	1.07E-10	1.09E-07	1.98E-05	0.000913	0.090699	0.87705	0.031314
F	0	0	0	0	0	0	1

Table 81: Stressed transition matrix for year 2011-Negative shock

2011	A+	A	B+	B	C	D	F
A+	0.78058	0.033271	0.027681	0.02316	0.035302	0.045665	0.054337
A	0.13567	0.23565	0.27731	0.20245	0.13561	0.013301	7.58E-06
B+	0.025692	0.11581	0.25255	0.2899	0.28104	0.034994	1.32E-05
B	0.001171	0.032035	0.20113	0.38636	0.36343	0.015875	2.54E-08
C	2.60E-08	1.75E-05	0.00164	0.032648	0.53279	0.43281	9.26E-05
D	1.06E-10	1.08E-07	1.97E-05	0.000911	0.090603	0.87711	0.031356
F	0	0	0	0	0	0	1

Table 82: Stressed transition matrix for year 2012-Negative shock

2012	A+	A	B+	B	C	D	F
A+	0.78129	0.033205	0.027618	0.023101	0.035202	0.045508	0.054072
A	0.13625	0.23607	0.27729	0.20208	0.13508	0.013211	7.49E-06
B+	0.025777	0.11605	0.25277	0.28985	0.28065	0.034885	1.31E-05
B	0.001259	0.033596	0.20624	0.38789	0.35599	0.015027	2.25E-08
C	2.68E-08	1.79E-05	0.001668	0.033023	0.53446	0.43074	9.07E-05
D	1.06E-10	1.08E-07	1.97E-05	0.000911	0.090565	0.87713	0.031373
F	0	0	0	0	0	0	1

Table 83: Stressed transition matrix for year 2013-Negative shock

2013	A+	A	B+	B	C	D	F
A+	0.78158	0.033178	0.027593	0.023078	0.035161	0.045445	0.053967
A	0.13648	0.23624	0.27728	0.20194	0.13487	0.013175	7.45E-06
B+	0.025811	0.11614	0.25286	0.28984	0.28049	0.034841	1.31E-05
B	0.001296	0.034238	0.20829	0.38846	0.35302	0.014698	2.14E-08
C	2.72E-08	1.80E-05	0.001679	0.033174	0.53513	0.42991	8.99E-05
D	1.06E-10	1.08E-07	1.97E-05	0.00091	0.090549	0.87714	0.031379
F	0	0	0	0	0	0	1

Table 84: Stressed transition matrix for year 2014-Negative shock

2014	A+	A	B+	B	C	D	F
A+	0.78169	0.033168	0.027583	0.023069	0.035145	0.04542	0.053925
A	0.13657	0.23631	0.27728	0.20188	0.13479	0.013161	7.44E-06
B+	0.025825	0.11618	0.2529	0.28983	0.28043	0.034824	1.31E-05
B	0.001311	0.034497	0.20912	0.38868	0.35183	0.014569	2.10E-08
C	2.73E-08	1.81E-05	0.001684	0.033234	0.5354	0.42958	8.96E-05
D	1.06E-10	1.08E-07	1.97E-05	0.00091	0.090543	0.87714	0.031382
F	0	0	0	0	0	0	1

Appendix E.7

The stressed transition matrices under positive shock to the factor over the period 2010 to 2014, under ”ceteris paribus” scenario are reported in Table 85-89.

Table 85: Stressed transition matrix for year 2010-Positive shock

2010	A+	A	B+	B	C	D	F
A+	0.81825	0.029479	0.024154	0.019926	0.029815	0.037333	0.041039
A	0.17063	0.25792	0.27336	0.18094	0.10814	0.009002	3.83E-06
B+	0.030822	0.12925	0.26483	0.28674	0.25907	0.029288	9.25E-06
B	0.034455	0.23596	0.42085	0.2459	0.062462	0.000374	1.26E-11
C	1.31E-07	6.07E-05	0.004069	0.059457	0.61531	0.32107	2.74E-05
D	9.74E-11	1.01E-07	1.86E-05	0.000871	0.088468	0.87833	0.03231
F	0	0	0	0	0	0	1

Table 86: Stressed transition matrix for year 2011-Positive shock

2011	A+	A	B+	B	C	D	F
A+	0.80818	0.030546	0.025136	0.020817	0.03131	0.039561	0.044451
A	0.16035	0.25204	0.27504	0.1871	0.11542	0.010058	4.64E-06
B+	0.029313	0.12543	0.2615	0.28779	0.26516	0.030796	1.02E-05
B	0.015371	0.15494	0.39249	0.32018	0.1158	0.001222	1.22E-10
C	8.44E-08	4.33E-05	0.003184	0.050713	0.59489	0.35113	3.88E-05
D	9.98E-11	1.03E-07	1.89E-05	0.000882	0.08906	0.878	0.032041
F	0	0	0	0	0	0	1

Table 87: Stressed transition matrix for year 2012-Positive shock

2012	A+	A	B+	B	C	D	F
A+	0.80405	0.030972	0.025529	0.021177	0.031916	0.040474	0.045878
A	0.15635	0.24961	0.27559	0.18953	0.11841	0.010509	5.01E-06
B+	0.028726	0.12391	0.26015	0.28817	0.26761	0.031417	1.06E-05
B	0.010811	0.12716	0.37061	0.34536	0.14416	0.001904	2.93E-10
C	7.06E-08	3.77E-05	0.002882	0.047503	0.58611	0.36343	4.46E-05
D	1.01E-10	1.04E-07	1.90E-05	0.000887	0.089297	0.87786	0.031935
F	0	0	0	0	0	0	1

Table 88: Stressed transition matrix for year 2013-Positive shock

2013	A+	A	B+	B	C	D	F
A+	0.80239	0.031142	0.025687	0.021321	0.03216	0.040842	0.046459
A	0.15477	0.24862	0.27579	0.1905	0.11962	0.010695	5.16E-06
B+	0.028495	0.12331	0.25961	0.28832	0.26859	0.031668	1.08E-05
B	0.009347	0.11694	0.3605	0.35427	0.15668	0.002262	4.13E-10
C	6.57E-08	3.57E-05	0.002768	0.046264	0.5825	0.36839	4.71E-05
D	1.01E-10	1.04E-07	1.91E-05	0.000889	0.089392	0.87781	0.031892
F	0	0	0	0	0	0	1

Table 89: Stressed transition matrix for year 2014-Positive shock

2014	A+	A	B+	B	C	D	F
A+	0.80172	0.03121	0.02575	0.021379	0.032258	0.04099	0.046693
A	0.15414	0.24823	0.27586	0.19089	0.12011	0.01077	5.23E-06
B+	0.028402	0.12307	0.25939	0.28838	0.26898	0.031769	1.09E-05
B	0.008812	0.113	0.35627	0.35763	0.16187	0.002422	4.74E-10
C	6.38E-08	3.49E-05	0.002724	0.045775	0.58104	0.37038	4.81E-05
D	1.01E-10	1.04E-07	1.91E-05	0.000889	0.089431	0.87779	0.031875
F	0	0	0	0	0	0	1

Appendix E.8

The stressed transition matrices under negative shock to the factor over the period 2010 to 2014, under "ceteris paribus" scenario are reported in Table 90-94.

Table 90: Stressed transition matrix for year 2010-Negative shock

2010	A+	A	B+	B	C	D	F
A+	0.78337	0.033009	0.027433	0.02293	0.034906	0.045048	0.053301
A	0.13796	0.23731	0.27722	0.201	0.13355	0.012949	7.23E-06
B+	0.026028	0.11673	0.25344	0.28973	0.27949	0.034565	1.29E-05
B	0.001553	0.038547	0.22147	0.39143	0.33424	0.012753	1.56E-08
C	2.93E-08	1.91E-05	0.001753	0.034145	0.53935	0.42465	8.53E-05
D	1.06E-10	1.08E-07	1.96E-05	0.000909	0.090452	0.8772	0.031422
F	0	0	0	0	0	0	1

Table 91: Stressed transition matrix for year 2011-Negative shock

2011	A+	A	B+	B	C	D	F
A+	0.79422	0.03196	0.026449	0.02202	0.033348	0.042649	0.049351
A	0.14728	0.24378	0.27659	0.19513	0.12558	0.011632	5.98E-06
B+	0.027396	0.12042	0.25695	0.28899	0.27334	0.032906	1.17E-05
B	0.004454	0.075093	0.30364	0.38559	0.22621	0.005011	2.10E-09
C	4.63E-08	2.72E-05	0.002274	0.040623	0.56436	0.39266	6.11E-05
D	1.03E-10	1.06E-07	1.93E-05	0.000897	0.089853	0.87754	0.031686
F	0	0	0	0	0	0	1

Table 92: Stressed transition matrix for year 2012-Negative shock

2012	A+	A	B+	B	C	D	F
A+	0.79847	0.031537	0.026055	0.021658	0.032732	0.041709	0.047837
A	0.15112	0.2463	0.27621	0.19275	0.12248	0.011139	5.55E-06
B+	0.02796	0.12191	0.25833	0.28865	0.27088	0.032261	1.12E-05
B	0.006587	0.095175	0.33445	0.37201	0.18842	0.003348	9.14E-10
C	5.55E-08	3.13E-05	0.002519	0.043469	0.57388	0.38004	5.34E-05
D	1.02E-10	1.05E-07	1.92E-05	0.000893	0.089615	0.87768	0.031792
F	0	0	0	0	0	0	1

Table 93: Stressed transition matrix for year 2013-Negative shock

2013	A+	A	B+	B	C	D	F
A+	0.80016	0.031368	0.025897	0.021513	0.032486	0.041336	0.047243
A	0.15268	0.2473	0.27603	0.19178	0.12125	0.010947	5.38E-06
B+	0.028188	0.12251	0.25888	0.28851	0.2699	0.032005	1.10E-05
B	0.007667	0.10414	0.346	0.36497	0.17439	0.002835	6.52E-10
C	5.96E-08	3.31E-05	0.002623	0.044651	0.57761	0.37503	5.06E-05
D	1.02E-10	1.05E-07	1.92E-05	0.000891	0.089519	0.87774	0.031835
F	0	0	0	0	0	0	1

Table 94: Stressed transition matrix for year 2014-Negative shock

2014	A+	A	B+	B	C	D	F
A+	0.80083	0.0313	0.025834	0.021456	0.032388	0.041188	0.047006
A	0.1533	0.2477	0.27596	0.1914	0.12076	0.01087	5.31E-06
B+	0.02828	0.12275	0.2591	0.28845	0.26951	0.031904	1.10E-05
B	0.00814	0.10788	0.35047	0.36191	0.16895	0.002651	5.69E-10
C	6.14E-08	3.39E-05	0.002666	0.04513	0.57908	0.37304	4.95E-05
D	1.02E-10	1.04E-07	1.91E-05	0.00089	0.089481	0.87776	0.031852
F	0	0	0	0	0	0	1

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