Causality between Returns and Traded Volumes

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Abstract

This paper examines causality between the series of returns and transaction volumes in high frequency data. The dynamics of both series is restricted to transitions between a finite number of states. Depending on the state selection criteria, this approach approximates the dynamics of varying market regimes, or in a broader sense reflects the time varying heterogeneity of traders behavior. Our analysis is based on returns and volumes represented by Markov chains with constant or time varying transition probabilities. We derive methods to estimate the transition probabilities, the long run equilibrium probability, and the instantaneous speed of adjustments. The limiting transition probability approximates the average proportion of time spent by the processes in a given state whereas the adjustment speed reveals the frequency of stock market fluctuations between states. The univariate return series is examined to identify varying market regimes and determine the impact of state specification on temporal dependence. In the bivariate framework we investigate comovements between volumes and transaction prices, and propose tests for Granger causality. The trade size threshold yielding a dichotomous process featuring maximum volume-price causality is proposed as a volume classification criterion. We apply our methods to the Alcatel stock data recorded in real and calendar time, and discuss implications of the sampling frequency.

Keywords: High Frequency Data, Price-Volume Relationship, Causality, Heterogeneity of Traders.

JEL numbers: C22, C32, G10, G12

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Résumé

Causalité entre rendements

et volumes de transactions.

Ce papier s'intéresse à la causalité entre rendements et volumes sur données haute-fréquence. La dynamique de ces séries est restreinte à un nombre fini d'états et les processus sont représentés par des chaines de Markov avec une matrice de transition qui peut être constante ou fonction du temps. Nous introduisons des procédures statistiques pour estimer les probabilités de transition, l'équilibre de long terme et la vitesse instantanée. La distribution stationnaire fournit des renseignements sur le temps moyen passé dans chaque état tandis que la vitesse d'ajustement révèle l'importance des fluctuations entre états. Dans le cadre de la spécification multivariée, nous analysons de facon précise les comouvements entre volumes et prix dans le but de mettre en évidence des régimes de transactions refletant l'hétérogénéité des investisseurs. Nous effectuons en particulier une analyse causale complète. L'approche est finalement appliquée aux données de cotations correspondant au titre Alcatel, considérées à la fois en temps de transactions et en temps calendaire, ceci permettant de discuter les effets de la fréquence de transactions.

Mots clés: Données haute-fréquence, relation prix-volume, causalité, hetérogeneité des investisseurs.

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1 Introduction

The empirical evidence documenting the relationship between prices and traded volumes is constantly growing. It has significantly progressed since the market breaks of 1987 and 1989 with episodes of high price volatility coupled with large trading volumes. More recently, the implementation of electronic trading systems provided new areas of investigation for applied research due to the accessibility of intraday data. In the literature we now find empirical studies examining the price-volume relationship in data sampled at various frequencies. There exist studies involving daily data [Tauchen, Pitts (1983), Karpoff (1987), Gallant, Rossi, Tauchen (1990), Lamoureux, Lastrapes (1994), Jones, Kaul, Lipson (1994)], hourly or half-hourly data [Mulherin, Gerety (1988), Foster, Viswanathan (1993)], and high frequency data recorded in real time at unequal intervals determined by trade arrivals [see, Jones, Kaul, Lipson (1995), Engle, Russell (1998), Darolles, Gourieroux, Lefol (1998)]. Although the empirical findings reported in these studies coincide to some extent, a formal ground for comparison has not been established yet and inference based on data observed at distinct sampling frequencies appears sometimes hard to reconcile. For this reason, researchers need to develop adequate procedures of temporal aggregation and learn to distinguish for example, the proportional variation of daily volume attributed to volumes of individual trades, from that due to the trade intensity [Engle, Russell (1997), Gourieroux, Jasiak, Lefol (1997)]. Another difficulty stems from the necessity to aggregate outcomes of distinct individual behaviors, and various strategies on a given trading day. The comparability of empirical results is also disturbed by the diversity of markets due to the co-existence of order driven markets equipped with electronic order matching devices [e.g the Paris Bourse, the Toronto Stock Exchange] and markets organized with market makers, like the New York Stock Exchange. On markets belonging to the last category we find intermediaries whose task is to ensure minimum liquidity. Their role on the order driven markets is automatically fulfilled by a specific type of order, called the market order [1]. It is executed at the best current market price and hence guarantees sufficient liquidity for orders queued in the middle of the order book. Moreover, there are selected investors who provide liquidity at prices differing substantially from the market price, and benefit in exchange from lower transaction costs. Among existing markets, we also distinguish markets involving single price clearing auctions which take place at fixed times, studied among others by Tauchen and Pitts (1983) and adopted as a general framework in a majority of theoretical papers, and continuous auctions markets.

Let us briefly summarize some "stylized facts" concerning the price-volume relationship docu-

[1] Note the difference between the definition of market order on the NYSE, and on an order driven market, like for example the Paris Bourse. In Paris, the "market order" is executed at the best current ask or bid price in the order book, up to the volume available for this price. The eventually outstanding volume remains in the queue as a limit order.

mented in the literature:

i) The expected returns depend on traded volumes. This dependence is interpreted in the literature as a liquidity risk premium, and represents a counterpart of the volatility risk premium. The negative correlation is confirmed by inference from simple linear regressions of returns on volumes, disregarding the volatility effect. Surprisingly, linear regressions involving explanatory variables other than volume produce contradictory evidence. It is hence unclear to what extent the negative sign of the regression coefficient on volume depends on the presence and type of other regressors. Equivalently the sign of the volatility risk premium depends on the information set available to econometricians. It seems that the presence of volume as an explanatory variable does not alter the sign of correlation. Indeed, Gallant, Rossi and Tauchen (1992) found a positive return-volatility relationship after introducing the lagged volume as a conditioning variable.

ii) The early empirical investigations were focused on the contemporaneous relationship between the absolute values of price changes as proxy for volatility and volumes, and documented a positive correlation between these variables. [see, e.g. Tauchen, Pitts (1983), and Karpoff (1987) for a survey]. In particular Lamoureux, Lastrapes (1991) showed that volume contains significant information to improve the prediction of price volatility.

iii) There also exist evidence for reverse effects. The prediction of intratrade durations, which partially determine the daily exchanged volume, is improved by conditioning on lagged prices or lagged volatilities. Such causal relationship has been systematically revealed by studies on high frequency data based on ACD-GARCH models, Autoregressive Multinomial models [Engle, Russell (1998)] or models involving the birth and death processes [Darolles, Gourieroux, Lefol (1998)].

iv) The dynamic relationship between prices and volumes is highly nonlinear, and can not be sufficiently explored by studies involving only the first and second order conditional moments. For example, the leverage effect, i.e. the asymmetric volatility response to positive and negative changes in prices, becomes substantially attenuated by conditioning on lagged volumes. The nonlinearity justifies the growing interest in nonparametric analysis of price and volume dynamics [Gallant, Rossi, Tauchen (1992)].

v) At a daily sampling frequency, the linear volatility-volume relationship may even disappear when the number of transactions is included in the set of conditioning variables [Jones, Kaul, Lipson (1995)]. It is not clear whether this result would hold if nonlinear patterns were accommodated, and data sampled at higher frequencies were considered.

Several of these issues were also addressed in theoretical contributions. From the theoretical perspectives, an analysis of the price-volume relationship has to be performed under the assumption of an incomplete market where the past prices do not convey all necessary information. This approach was adopted for prices and volumes examined in the market equilibrium framework by

Admati, Pfeiderer (1988), Tauchen, Pitts (1983), Huffman (1987, 1988), Ketterer, Marcet (1989), Wang (1994) and He and Wang (1992). These authors introduced a sequence of equilibria [or only one equilibrium] occurring at some predetermined times. It has to be emphasized that this setup corresponds to a multiple clearing auction market and not to a continuous matching process. At equilibrium the only incentive for investors to trade is their heterogeneity in terms of available information, investment opportunities or tax constraints [Constantinides (1984)]. The prices and volumes are determined by matching the demand and supply at fixed dates. Both the equilibrium prices and volumes depend on the degree of heterogeneity which creates the perceived relationship between these variables.

A similar argument of heterogeneity driven market may be put forward in the context of continuous auction markets. There exist several yet unexplored issues concerning the temporal dependencies of the order queue dynamics i.e. the impact of the market history on the arrivals and cancellations of orders, the prices, volumes and times of trades. A microstructure dynamic model representing these phenomena has not yet been proposed. We outline below some insights on the volume-price relationship provided by the microstructure theory.

Initially, market liquidity was considered as a factor generating the price-volume relationship. Consider a stock which is not highly ranked in terms of liquidity. A transaction involving significant volume, even not motivated by some specific information release, may affect the price of this stock. In general this effect is instantaneous and triggers a price decline if the trade is initiated by the supply side of the market or a price increase otherwise. The magnitude of price increments depends on the slopes of bid and ask functions showing the unitary prices of volume for the priority to be served instantaneously. For example, Black (1971) assumed that an immediate block trade results in a price movement proportional to the size of the block and interpreted it as an opportunity cost. Consequently, one would expect this cost to be included in the expected return, creating the aforementioned liquidity premium. In intermediate term we can expect to see a negative relationship between the expected return and some liquidity measures, like for example the daily volume. However the instantaneous and intermediate term effects may only be partially observable for various reasons. Since the first lines of the order book are nowadays revealed through electronic trading systems, an investor may potentially choose to place his/her order when the depth of these first lines, i.e. the queued volume is sufficiently large. However this strategy is only admissible if the investor is not required to trade before a fixed deadline. This is why it may be useful to distinguish between the patient investors who provide liquidity and the rushed investors who demand it [see e.g. Harris (1995), Handa, Schwartz (1996)].

Numerous microstructure models analyze the trading dynamics in the framework of asymmetric information. The common assumption presumes the existence of investors who are better informed than the rest of market participants, including other investors and market makers, given the time and environment. The well informed traders try to take advantage of their privileged information by selecting appropriate trading strategies. However their actions are observed by the market makers, or even by all remaining investors on the order driven markets, through the revealed order books. Hence, they can infer from the behavior of informed investors about the content of their private information. The majority of theoretical studies focus on the price discovery process ,i.e. the joint dynamics of trading prices and spreads, and suppose fixed trading dates and traded volumes. Moreover, in general three types of protagonists are considered: the informed traders, the uninformed ones and the market makers, and therefore such approach does not represent the dynamics of order driven markets with electronic matching procedures. Among models falling into this category we find Kyle (1985), Glosten, Milgrom (1985), Admati, Pfeiderer (1988, 1989), Grundy, McNichols(1989), Foster, Viswanathan (1990, 1993), Blume et al. (1994). The argument of asymmetric information may also hold in a price-volume analysis. An investor who believes that the current price significantly underestimates the fundamental value will attempt to buy a volume of shares depending on the size of the underestimation bias, the slopes of the bid and ask functions and the transaction costs. Therefore, by introducing the size of trade variable we end up facing an adverse selection problem in the context of security trading [Pfeiderer (1984), Kim, Verecchia (1991)]. Our trader's behavior suggests a positive relationship between absolute price changes and traded volumes. He may eventually camouflage his activity by trading sequentially small volumes instead of executing one large transaction. This last strategy could attenuate the positive sign of the relationship, although the final outcome would depend on the proportion of uninformed traders [Holden, Subrahmanyan (1992)].

Information and liquidity, the key factors driving trades on financial markets can create joint effects, discussed by Admati, Pfeiderer (1988). The authors distinguish between nondiscretionary liquidity traders who have to trade at a particular time, and discretionary liquidity traders who are free to choose the time of trade. Admati and Pfeiderer found that if acquisition of information is endogenous, in equilibrium more traders become informed in periods of concentrated liquidity trading and prices are more informative.

The various approaches to price-volume relationship described above rely on a common assumption of heterogeneous traders, who may be time constrained or not, supplying or consuming liquidity, and finally, informed or not. Can we substitute for the price-volume analysis a study of the dynamics of these heterogeneous groups? A major difficulty involved in this task is the nonobservability of traders' identities and characteristics contrary to prices and volumes. Still we can learn about the latent heterogeneity by investigating the joint behavior of returns and volumes. This is the objective of our paper, where we explain how to identify different endogenous trading regimes. These regimes are determined by significant downwards and upwards price movements and small and large traded volumes. The core of the paper constitutes the selection of price and volume thresholds defining the limits between the bear and bull markets and small and large volumes, respectively, called the optimal state selection.

The paper is organized as follows: in section 2 we consider an univariate series of stock prices and a dichotomous qualitative process characterizing the price dynamics. The qualitative process is next modeled by a two state Markov chain with a transition matrix parametrized alternatively by probabilities of one step transitions, and by the adjustment speed and the long run equilibrium probability. The parameters can consistently be estimated using methods related to the standard regression analysis. This section also covers state selection yielding uncorrelated qualitative price processes. The qualitative bivariate Markov specification provides a convenient framework for inference on nonlinear causal links between prices and volumes. In section 3 we consider the corresponding transition probability matrix and determine the relationship between the multivariate state transition probabilities and the coefficients of a multivariate SUR model. Next, we propose causality tests involving linear regression coefficients. We finally discuss the optimal state selection criteria. Among them we consider thresholds maximizing the causal relationship between volume and returns in a two state process. The empirical results are presented in section 4 where we analyse high frequency data for the Alcatel stock traded on the Paris Bourse. Two sampling schemes, corresponding to the transaction and calendar time, respectively are examined. Section 5 concludes the paper. Technical details are presented in the Appendix.

2 Univariate Dynamic Patterns

In this section we investigate the dynamics of a Markov chain (Z_t) making transitions between two states 0 and 1. For ease of exposition, we consider the Markov process of order one, although the results and interpretations can easily be extended to any higher order processes.

2.1 Financial Applications

The two state framework may be used in various ways to analyze the trading process, i.e. stock prices and volumes. The states are distinguished according to some dichotomous qualitative features of the quantitative series of prices and volumes. For example we can consider:

a) the direction (increase or decrease) of the price (or log-price) evolution:

$$Z_t = \begin{cases} 1, & \text{if } log(p_t) - log(p_{t-1}) > 0, \\ 0, & \text{otherwise.} \end{cases}$$

b) the comparison of price modification with a given threshold not necessarily equal to zero:

$$Z_t(c) = \begin{cases} 1, & \text{if } log(p_t) - log(p_{t-1}) > c, \\ 0, & \text{otherwise.} \end{cases}$$

This approach can be applied to compare the price evolution with the behavior of the riskfree asset, by choosing $c = \log(1 + r)$, where r is the riskfree rate.

There exist other possibilities of threshold selection which may also be considered for financial analysis. For instance, we may fix c at a very large value [resp. small value] in an analysis of the dynamics of positive [resp. negative] extreme returns. This state specification can further be adapted to a multistate setup where, for example, several price modifications are distinguished.

c) The qualitative analysis may also be used to study the dynamics of the trade initiating side of the market by verifying if the observed trading prices correspond to asks or bids [Hedvall, Rosenquist (1997)]:

$$Z_t = \begin{cases} 1, & \text{if } p_t = ask_t, \\ 0, & \text{if } p_t = bid_t. \end{cases}$$

d) Some other qualitative features may concern joint trading, for example involving substitution between an underlying asset and a derivative:

$$Z_t = \begin{cases} 1, & \text{if both asset and option are traded between } t \text{ and } t + \Delta t, \\ 0, & \text{otherwise,} \end{cases}$$

where Δt is a predetermined time interval.

e) Finally we may also examine some features of the volume series and, for example distinguish trades of small and large sizes:

$$Z_t = \begin{cases} 1, & \text{if } v_t > c, \\ 0, & \text{otherwise.} \end{cases}$$

2.2 Parametrization of the Transition Matrix

The one step transition probabilities of Z_t are:

$$P(Z_t = i \mid Z_{t-1} = j) = \Pi_{ij},$$

where $i = 0, 1$ and $j = 0, 1$.

The transition matrix is the 2×2 matrix:

$$\Pi = \begin{pmatrix} \Pi_{00} & \Pi_{01} \\ \Pi_{10} & \Pi_{11} \end{pmatrix},$$

where: $\Pi_{00} + \Pi_{10} = \Pi_{01} + \Pi_{11} = 1,\ \Pi_{ij} \geq 0, \forall i, j.$

The transition matrix Π has two eigenvalues: one of them is equal to 1, the other one is:

$$\lambda \;=\; \Pi_{00} + \Pi_{11} - 1 \;=\; \Pi_{11} - \Pi_{10},$$

with a modulus less than 1.

The limiting (stationary) probability distribution is:

$$\pi_1 = P(Z_t = 1) = \frac{\Pi_{10}}{\Pi_{01} + \Pi_{10}}, \quad \pi_0 = P(Z_t = 0) = \frac{\Pi_{01}}{\Pi_{10} + \Pi_{01}}.$$

Clearly, the two parametric representation of the Markov chain dynamics involving (Π_{10}, Π_{11}) or (λ, π_1) are equivalent. We have indeed the following equality:

$$\Pi \;=\; Id + (1-\lambda)(-1,1)'(\pi_1, -\pi_0).$$

2.3 Regression Interpretation of Transition Probabilities

The transition probabilities, the eigenvalue and the stationary distribution can be interpreted as regression coefficients. Indeed the prediction of Z_t given the information available at $t-1$ is:

$$\begin{aligned} E[Z_t \mid Z_{t-1}] &= P[Z_t = 1 \mid Z_{t-1}] \\ &= \Pi_{11} Z_{t-1} \,+\, \Pi_{10}(1 - Z_{t-1}). \end{aligned} \tag{2.1}$$

Therefore, if we consider the linear regression:

$$Z_t = aZ_{t-1} + b(1 - Z_{t-1}) + u_t, \; E[u_t \mid Z_{t-1}] = 0, \tag{2.2}$$

or equivalently the linear regression with a constant term:

$$Z_t = \alpha Z_{t-1} + b + u_t, \, E[u_t \mid Z_{t-1}] = 0,$$

we deduce that

$$a = \Pi_{11}, \, b = \Pi_{10},$$

and

$$\alpha \;=\; \Pi_{11} - \Pi_{10} \;=\; \lambda.$$

The coefficient α can be interpreted as an adjustment speed and coincides with the second eigenvalue. Moreover the previous regression may also be written :

$$Z_t - m \;=\; \alpha(Z_{t-1} - m) + u_t,\; E[u_t \mid Z_{t-1}] = 0, \tag{2.3}$$

where $m = E[Z_t] = P[Z_t = 1] = \pi_1$ is the marginal probability of state 1.

The regression-based estimation of transition probabilities is straightforward. The least squares estimators $\hat{a}$ and $\hat{b}$ are given by:

$$\begin{aligned}\begin{pmatrix}\hat{a}\\ \hat{b}\end{pmatrix} &= \begin{pmatrix}\sum Z_{t-1}^2 & \sum Z_{t-1}(1-Z_{t-1})\\ \sum Z_{t-1}(1-Z_{t-1}) & \sum(1-Z_{t-1})^2\end{pmatrix}^{-1}\begin{pmatrix}\sum Z_t Z_{t-1}\\ \sum Z_t(1-Z_{t-1})\end{pmatrix}\\ &= \begin{pmatrix}T_1 & 0\\ 0 & T_0\end{pmatrix}^{-1}\begin{pmatrix}T_1\,\hat{\Pi}_{11}\\ T_2\,\hat{\Pi}_{10}\end{pmatrix} = \begin{pmatrix}\hat{\Pi}_{11}\\ \hat{\Pi}_{10}\end{pmatrix},\end{aligned}$$

with T_1 being the number of observations for which $Z = 1$ and T_0 the number of observations for which $Z = 0$. The least squares estimators are equal to the empirical frequencies, and coincide with the maximum likelihood (ML) estimators of Π_{11} and Π_{10}. Hence, the ML estimators can be obtained by the least squares applied to regression (2.2) or (2.3) using standard software.

2.4 The Uncorrelated States Specification

In example b) of qualitative processes representing returns, we defined a family of Markov chains indexed by the threshold c. At this point we have to emphasize that the Markov property is not invariant with respect to the selected c and, in general, a Markov chain $Z_t(c_0)$ does not remain Markov for a different state selection threshold $c \neq c_0$. For this reason it is necessary to identify all possible c yielding processes featuring the desired properties. Markov chains represent several advantages for empirical analysis due to their simple temporal dependence structure and predictability.

In this section we study the dynamics of the chain $(Z_t(c) = 1_{y_t > c})$ and its dependence on the selected c. In particular, we investigate to what extent the serial correlation of the underlying series is altered by imposing a qualitative representation. There exists empirical evidence suggesting that even if returns exhibit some temporal dependence when analyzed as a quantitative process, serial correlation may disappear in a qualitative series of sign change indicators.

In the first step we specify the relationship between the threshold c and the first order autocorrelation as a function of c. For applied work we introduce the empirical first order autocorrelation $\hat{\rho}_T(c)$. We define the threshold estimator:

$$\tilde{c}_T = \min_c \hat{\rho}_T(c)^2,$$

which approximates the c value providing the lowest absolute value of the first order autocorrelation. Since the estimated first order autocorrelation is a stepwise function of c, the solution $\hat{c}_T$ may not be unique or appear as a solution of the first order conditions. Hence, it may be useful to introduce a smoothed first order autocorrelogram involving the empirical autocorrelations $\hat{\rho}_{T,k}(c)$ of $Z_t(k;\, c) = F\left(\frac{Y_t - c}{k}\right)$, where F is a given c.d.f., for example a standard normal, and k is a bandwidth. The corresponding estimator of minimal correlation threshold is:

$$\hat{c}_{T,k} = \min_c \hat{\rho}^2_{T,k}(c).$$

The following property is proved in the Appendix:

Property 1:

Let (Y_t) be a strongly stationary process with a continuous density function and consider a qualitative process defined by $Z_t(c) = 1_{Y_t \geq c}$. Under regularity conditions given in the Appendix and if the theoretical first order autocorrelation

$$c \to \rho(c)$$

admits a unique zero first order autocorrelation threshold c_1, then for $T \to +\infty$ and $k \to 0$ at an appropriate rate:

i) $\hat{c}_{T,k}$ is a consistent estimator of c_1;

ii) For T sufficiently large $\hat{\rho}_{T,k}[\hat{c}_{T,k}] = 0$.

iii) $\hat{c}_{T,k}$ is asymptotically normal, with

$$Var_{asy}(\sqrt{T}(\hat{c}_{T,k} - c_1)) =$$

$$\left\{ f(c_1) \frac{P(Y_{t-1} > c_1 | Y_t = c_1) + P(Y_t > c_1 | Y_{t-1} = c_1) - 2P(Y_t > c_1)}{P(Y_t > c_1)(1 - P(Y_t > c_1))} \right\}^2 ,$$

where $f(.)$ is the marginal density of Y_t.

The second condition can be used to detect the existence of a zero first order autocorrelation threshold c_1. Indeed if the zero autocorrelation threshold does not exist, the minimum of ρ^2 is strictly positive and the same holds asymptotically for $\hat{\rho}^2_{T,k}[\hat{c}_{T,k}]$.

This approach may be directly extended to accommodate autocorrelations of higher orders by estimating thresholds:

$$\hat{c}_{T,k}(h) = \min_c \hat{\rho}_{T,k}(h;\, c),$$

where $\hat{\rho}_{T,k}(h;c)$ is the smoothed autocorrelation of order h of $(Z_t(c))$. In the next step, the estimated thresholds $\hat{c}_{T,k}(h)$ may be compared. For example, the existence of a threshold c^* such that $\hat{c}_{T,k}(h) \approx c^*$, $\forall h \geq 1$ and $\hat{\rho}_{T,k}[c^*]$ are close to zero for $h \geq 1$ is a strong evidence in favor of $(Z_t(c))$ being a white noise.

3 Causality Tests

The Markov chain approach can be extended to a bivariate setup for a joint analysis of returns and volumes. In this section we study causal relations between the series and develop various inference methods. For clarity of exposition, the assumption of Markov processes of order one is maintained.

We introduce thresholds distinguishing positive and negative signs of price changes, and large and small volumes:

$$Z_t(a) = \begin{cases} 1, & \text{if } \log(p_t) - \log(p_{t-1}) > a, \\ 0, & \text{otherwise,} \end{cases}$$

$$Y_t(c) = \begin{cases} 1, & \text{if } v_t > c, \\ 0, & \text{otherwise.} \end{cases}$$

and consider the qualitative bivariate process $[Y_t(c),\ Z_t(a)]'$ of returns and volumes. The transition probabilities defined by:

$$P(Z_t = i, Y_t = j \,|\, Z_{t-1} = k, Y_{t-1} = l) \;=\; \Pi_{ij|kl}, \quad i,j,k,l = 0,1$$

form a matrix with eigenvalues $1, \lambda_1, \lambda_2, \lambda_3$. By analogy to the univariate approach, we denote the stationary probabilities by $m_{11}, m_{10}, m_{01}, m_{00}$.

3.1 A SUR Representation of the Markov Chain

The bivariate model, like its univariate analogue discussed in section 2, can be equivalently represented by regressions. Consider the following SUR model [Zellner (1962)]:

$$\begin{aligned}
Z_tY_t &= \beta_{11} + \alpha_{11|11}Z_{t-1}Y_{t-1} + \alpha_{11|10}Z_{t-1}(1-Y_{t-1}) + \alpha_{11|01}(1-Z_{t-1})Y_{t-1} + u_{11t}, \\
Z_t(1-Y_t) &= \beta_{10} + \alpha_{10|11}Z_{t-1}Y_{t-1} + \alpha_{10|10}Z_{t-1}(1-Y_{t-1}) + \alpha_{10|01}(1-Z_{t-1})Y_{t-1} + u_{10t}, \\
(1-Z_t)Y_t &= \beta_{01} + \alpha_{01|11}Z_{t-1}Y_{t-1} + \alpha_{01|10}Z_{t-1}(1-Y_{t-1}) + \alpha_{01|01}(1-Z_{t-1})Y_{t-1} + u_{01t}.
\end{aligned}$$

The matrix representation of the system of equations is:

$$\begin{pmatrix} Z_tY_t - m_{11} \\ Z_t(1-Y_t) - m_{10} \\ (1-Z_t)Y_t - m_{01} \end{pmatrix} = A \begin{pmatrix} Z_{t-1}Y_{t-1} - m_{11} \\ Z_{t-1}(1-Y_{t-1}) - m_{10} \\ (1-Z_{t-1})Y_{t-1} - m_{01} \end{pmatrix} + u_t, \tag{3.4}$$

where $E(u_t|Z_{t-1}, Y_{t-1}) = 0$. We obtain a new parametrization of the transition matrix in terms of limiting probabilities (3 parameters) and the matrix of adjustment speed coefficients A (9 parameters). The eigenvalues of A are denoted by $\lambda_1, \lambda_2, \lambda_3$. Again, like in the univariate case, the maximum likelihood estimation of transition probabilities is equivalent to the least squares applied to the SUR model.

3.2 The Noncausality Hypotheses

Let us now consider the noncausality hypotheses. By definition [Geweke (1982),Granger (1969)] Y does not cause Z_t if the conditional density of Z_t given Y_{t-1}, Z_{t-1} is equal to the density of Z_t given Z_{t-1} only. This condition can be transformed into a set of linear constraints involving the parameters of the SUR model. From this specification we deduce:

$$Z_t = \beta_{11} + \beta_{10} + (\alpha_{11|11} + \alpha_{10|11})Z_{t-1}Y_{t-1} + (\alpha_{11|10} + \alpha_{10|10})Z_{t-1}(1 - Y_{t-1}) +$$
$$(\alpha_{11|01} + \alpha_{10|01})(1 - Z_{t-1})Y_{t-1} + u_{11t} + u_{10t}.$$

When $Y_{t-1} = 1$ the deterministic part of the model is:

$$\beta_{11} + \beta_{10} + (\alpha_{11|01} + \alpha_{10|01}) + (\alpha_{11|11} + \alpha_{10|11} - \alpha_{11|01} - \alpha_{10|01})Z_{t-1},$$

while for $Y_{t-1} = 0$ the expression simplifies to:

$$\beta_{11} + \beta_{10} + (\alpha_{11|10} + \alpha_{10|10})Z_{t-1}.$$

The null hypothesis of noncausality can be written:

$$H^0_{Y \to Z} : \{\alpha_{11|01} + \alpha_{10|01} = 0,\ \ \alpha_{11|11} + \alpha_{10|11} - \alpha_{11|10} - \alpha_{10|10} = 0\}.$$

Similarly, other noncausality hypotheses may also be written in terms of regression parameters. Thus, the null hypothesis of noncausality from Z to Y is:

$$H^0_{Z \to Y} : \{\alpha_{11|10} + \alpha_{01|10} = 0,\ \ \alpha_{11|11} + \alpha_{01|11} - \alpha_{11|01} - \alpha_{01|01} = 0\},$$

while the null hypothesis of instantaneous noncausality between Z and Y is satisfied when the deterministic part of Z_tY_t is the product of the deterministic parts of Z_t and Y_t. We deduce four constraints which define this null hypothesis $H^0_{Z \leftrightarrow Y}$:

$$\begin{aligned}
&\beta_{11} - \alpha_{11|11} - \alpha_{11|10} - \alpha_{11|01} = \\
&\qquad \{\beta_{11} + \beta_{01} - (\alpha_{11|11} + \alpha_{01|11}) - (\alpha_{11|10} + \alpha_{01|10}) - (\alpha_{11|01} + \alpha_{01|10})\} \\
&\qquad \{\beta_{11} + \beta_{10} - (\alpha_{11|11} + \alpha_{10|11}) - (\alpha_{11|10} + \alpha_{10|10}) - (\alpha_{11|01} + \alpha_{10|01})\},
\end{aligned}$$

$$\begin{aligned}
\alpha_{11|11} &= (\alpha_{11|11} + \alpha_{01|11})(\alpha_{11|11} + \alpha_{10|11}), \\
\alpha_{11|10} &= (\alpha_{11|10} + \alpha_{01|10})(\alpha_{11|10} + \alpha_{10|10}), \\
\alpha_{11|01} &= (\alpha_{11|01} + \alpha_{01|01})(\alpha_{11|01} + \alpha_{10|01}).
\end{aligned}$$

3.3 Additional Regressions for Causality Analysis

In the previous subsection, the nonlinear causality analysis of a bivariate Markov chain was based on regressions involving nonlinear qualitative regressors. We show below supplementary regressions corresponding to each type of causality. This approach greatly simplifies the computation of test statistics.

i) Unidirectional causality from Z to Y:

Let us consider the regression:

$$Y_t = \beta_{1.} + \alpha_{1.|1.}Y_{t-1} + \alpha_{1.|.1}Z_{t-1} + \alpha_{1.|11}Y_{t-1}Z_{t-1} + u_{1.,t}.$$

The hypothesis of noncausality from Z to Y corresponds to the constraints: $H^0_{Z\to Y} = \{\alpha_{1.|.1} = \alpha_{1.|11} = 0\}$. Interestingly, if the hypothesis is rejected we can find out if this outcome is due to the presence of uniquely linear dependencies. For this purpose, we can test if $\alpha_{1.|11} = 0$ and $\alpha_{1.|.1} \neq 0$.

ii) Unidirectional causality from Y to Z.

The regression to consider is symmetric to the previous one:

$$Z_t = \beta_{.1} + \alpha_{.1|1.}Y_{t-1} + \alpha_{.1|.1}Z_{t-1} + \alpha_{.1|11}Y_{t-1}Z_{t-1} + u_{.1,t},$$

and the null hypothesis of noncausality is :

$$H^0_{Y\to Z} = \{\alpha_{.1|1.} = \alpha_{.1|11} = 0\}.$$

iii) Instantaneous causality between Y and Z.

The instantaneous noncausality concerns the absence of influence of the current value of Z_t in the conditional distribution of Y_t given Z_t, Y_{t-1}, Z_{t-1}. The regression corresponding to the conditional distribution contains eight regressors:

$$Y_t = \gamma_1 + \delta_{1..} Z_t + \delta_{.1.} Y_{t-1} + \delta_{..1} Z_{t-1} + \delta_{11.} Z_t Y_{t-1} + + \delta_{1.1} Z_t Z_{t-1} + \delta_{.11} Y_{t-1} Z_{t-1} + \delta_{111} Z_t Y_{t-1} Z_{t-1} + v_{1,t}.$$

The null hypothesis is:

$$H^0_{Z \leftrightarrow Y} = \{\delta_{1..} = \delta_{11.} = \delta_{1.1} = \delta_{111} = 0\}.$$

A rejection may be due either to the lack of a linear relation of Y_t with Z_t, or to interactions of order two, or of order three between the variables.

3.4 Volume State Specification and Causality

In the causality analysis presented above we assumed that the threshold defining the states of the qualitative volume process is fixed and set, for example at the average volume per trade. Intuitively, we might expect that the state specification matters in causality inference. The volume threshold selection is ultimately oriented towards distinguishing two types of traders revealed by the size of their trades. Accordingly we focus on two groups of investors trading large or small volumes, respectively. The microstructure theory suggests that the subpopulations of traders, like the informed and uninformed ones are homogeneous in terms of their strategies concerning the prices and volumes of individual trades. This result justifies a simplifying assumption of the consistency of strategies within subgroups we introduce below for further analysis. Our final approach can be summarized as follows:

Under the assumption on the consistency of traders behavior, we discriminate between the two subpopulations of traders by searching the limiting value c providing the strongest causality from volume to price. We outline below one among several possible formal procedures: For each level of c, we compute the chi-square test statistics $\xi(c)$ for testing the hypothesis of noncausality from volume to returns. We know that $\xi(c)$ is a measure of the strength of the causal relation [Gourieroux, Monfort, Renault (1987)]. Hence the threshold estimate:

$$\hat{c}_T = \underset{c}{Argmax}\, \xi(c),$$

(or a smoothed version of this estimator) can be used as the discriminating level.

4 Empirical Results

We examine returns and volumes of trades of the Alcatel stock, recorded on the Paris Stock Exchange (Paris Bourse) in July and August 1996 (Figure 4.1). Prior to estimation, the opening trades [the simultaneous trades concealing split orders resp.] were deleted [aggregated resp.]. The data set consists of 20405 observations on returns ($\Delta \log(p_t)$) and volume by trade ($v_t = \log(V_t)$). The first series has sample mean -9.28E-6, variance 9.885E-7, and takes values from the interval [-0.0083, 0.0064]. The volume series has sample mean 6.6516, variance 88.5538, and observations varying in the interval [0.0000, 307.8432].

We proceed in two steps by investigating separately the data in transaction and calendar times. A unitary increment in transaction time is set by a trade arrival, disregarding the length of the waiting time between transactions. Conventionally, a unitary increment in calendar time corresponds to an integer multiple of one minute and may eventually be arbitrarily selected, depending on the sampling frequency of the data. In our analysis we use a 1 minute grid which roughly corresponds to an average duration between arrivals of Alcatel trades (52.56 sec). The comparison of results obtained from the calendar time and real time data is expected to reveal the effect of the frequency of trades, which is one of liquidity determinants.

4.1 Analysis in Trading Time

(i) Univariate analysis for fixed price and volume thresholds.

We consider the qualitative return series defined by the sign of price modification $Z_t(0) = 1_{\Delta \log p_t \geq 0}$, where t is the trade index, and the volume series $Y_t(c) = 1_{v_t \geq c}$, where v_t denotes the volume observed at trade t and c is the sample mean.

There are 13946 observations on non-negative returns and 6458 on strictly negative ones. The estimated stationary probability of the "up" state $\hat{m}$ is 0.6834 with standard error 0.0030, while the estimated speed adjustment coefficient $\hat{\alpha}$ is -0.0783 with standard error 0.0069. The negative sign of adjustment speed indicates the presence of fluctuations in the returns and is likely related to the so-called bid-ask bounce. Indeed, the trades can be initiated by either demand or supply side of the market resulting in an upcoming trading price equal to an ask or a bid price. To some extent, fluctuations of transaction prices, and therefore return fluctuations are due to the dynamics of ask or bid price selection process.

The state defining threshold in the volume series is fixed at the sample mean 6.6516. The volume is above or at the average level in 5952 transactions, and strictly below in 14452. The estimates of m and α are 0.2917 and 0.1054, with standard errors 0.0035 and 0.0069 respectively. We note that the estimator $\hat{m}$ is far below 0.5 indicating that the average volume traded by tick is far less than the median 4.6051. Hence, in the long run there is less probability of occurrence of state 1 than 0.

Qualitative data also reveal time variation of transition probabilities. We repeat the estimation of the m and α coefficients over consecutive hourly intervals. Figures 4.2 a and b present the trajectory of the $\hat{m}$ parameter in the return and volume series together with their confidence intervals, whereas Figures 4.3 a and b display the dynamics of $\hat{\alpha}$ in both series. To detect eventual intraday periodic patterns, we averaged the results over all days for each of the seven hours of a working day. The results are collected in Tables 4.1 and 4.2. (upper panels). Let us first discuss the return data. As expected, the long run parameter m is less varying than the speed parameter, and may be considered as time homogeneous except for trades before the market closure. Concerning the speed parameter, we observe some classical intraday seasonality, with higher absolute values recorded after the opening or during the lunch period, especially shortly before the opening of the London Stock Exchange (the period 11-12), or NYSE (the period 13-14) [see, Gourieroux, Jasiak, LeFol (1998)]. In the volume series, the intraday seasonality is more pronounced due to larger trades occurring in the morning and in the afternoon, while heterogeneity in traded volumes increases between 12-13. We find that over this period, the adjustment speed is negative. Surprisingly, in the first hour of NYSE activity $\hat{\alpha}$ is in absolute value almost twice of its value during the rest of the day.

(ii) Correlation analysis and state selection for return series

Figure 4.4 shows the impact of the threshold selection on the autocorrelation pattern of returns. The panels 1:4, beginning with the top one, display the autocorrelograms of quantitative series and three qualitative series corresponding to various thresholds equal to the mean, which is close to zero, the 10th and 95th percentiles, respectively. As expected the first order autocorrelation is significantly negative due to the bid-ask bounce. Beyond the bid-ask effect, the higher order autocorrelations in the quantitative series are not significant. This explains the random walk behavior observed in data sampled at lower, constant frequencies. We find that the autocorrelation patterns of qualitative and quantitative processes differ substantially for thresholds different from 0, especially a longer range of persistence is observed in large returns. The autocorrelograms of the quantitative process and the qualitative process with zero threshold are quite similar.

It is clear that the autocorrelation size as well as the range of persistence vary when thresholds are modified. Figure 4.5a displays the autocorrelation of order one in the return series as a function of the threshold varying between the sample minimum and maximum. The behavior of higher order correlations is illustrated in Figure 4.5b by the statistic $Q^* = \sum_{i=2}^{10} \hat{\rho}^2$, estimated again for thresholds varying between the sample minimum and maximum.

We note that zero is a particular threshold for the return series, since the statistics $\hat{\rho}_1$ and Q^* are approximately even functions. Moreover the portmanteau statistic Q^* is close to zero for $c = 0$, whereas the first order autocorrelation is at maximum in absolute value in a neighborhood of 0. This confirms the interpretation of Figures 4.4 and justifies the choice of zero threshold for the returns, as it yields a qualitative process with dynamics as close as possible to the dynamics of the quantitative data.

(iii) Causality analysis and state selection for volume series

There is a strong evidence of linear causality in the quantitative representations of return and volume processes already known in the empirical literature. The null hypothesis of absence of unidirectional causality is rejected by standard t-tests of coefficients in linear regressions of returns (volume) on lagged returns and volume. The estimated regressions are:

$$
\begin{aligned}
r_t &= 0.000013 - \underline{0.3862} r_{t-1} - \underline{0.000004} v_{t-1} \\
v_t &= \underline{5.8684} - \underline{160.3382} r_{t-1} + \underline{0.1175} v_{t-1} \\
v_t &= \underline{5.8775} - \underline{672.1791} r_t - \underline{419.9977} r_{t-1} + \underline{0.1148} v_{t-1}
\end{aligned}
$$

where all coefficients associated with the unidirectional causality hypotheses are significant (underlined) . As well, we reject the null hypothesis of absence of linear instantaneous causality in a regression of volume on returns and past volume and returns.

In the qualitative data defined by thresholds set at zero and the sample average for returns and volumes, respectively, the inference on unidirectional and instantaneous causality is based on the regression method (section 3.2), and yields similar results [see Table 4.3]. Our analysis also reveals significant variation of causality directions in time [Richard (1978)]. Figure 4.6 presents the p-values of unidirectional causality tests repeated over daily subsamples. The horizontal line at 0.05 separates the noncausality region (below) and the causality region (above). The two types of unidirectional causality coexist on some days, but no one seems to be significantly more frequent than the other.

The presence or absence of causality in the entire sample as well as its daily dynamics depend on the selected volume threshold. We illustrate this effect by plotting the values of the F-statistics for volume threshold levels varying between the sample minimum and the 99.94th percentile in Figure 4.7. The horizontal line at 2.99 corresponds to the critical value of the F-test for the noncausality hypothesis. Within this interval we find the maximal causality from volume to returns for a discriminating level of approximately 8.28, which exceeds the average volume. We also note that for this value (close to the median of volume but less than the mean) causality from returns to volume is almost eliminated. Therefore this threshold is a good candidate for discriminating among two categories of traders (see, section 3.4).

4.2 Analysis in Calendar Time

The literature suggests that the dynamic properties in trading and calendar times may differ due to time deformation [see e.g. Ghysels, Gourieroux, Jasiak (1998)]. It has also been observed that correlations in returns are, in general, weaker in trading time than in calendar time. We will see that this decrease in correlations is no longer observed when volumes are also taken into account.

(i) Univariate analysis for fixed price and volume thresholds.

The calendar time sample consists of 9813 observations. The empirical first two moments are slightly modified compared to real time data. The returns have mean -2.87E-7, variance 1.079E-6, while the volume's average is 6.4429 and variance 80.2347.

There are 6709 observations on returns in state 1 and 3103 on returns in state 0. The estimated $\hat{m}$ is 0.6837 with standard error 0.0043, while $\hat{\alpha}$ is still negative, -0.0670 with standard error 0.0100.

The volume is classified in two states using the threshold 6.4429, marginally lower than in real time data. The number of visits in state 1 is 2778, and 7034 in state 2. The coefficient $\hat{m}$ is 0.2831, (still far below 0.5) with standard error 0.0047, while $\hat{\alpha}$ is estimated at 0.0484 with standard error 0.0100.

Compared to real time, the stationary probabilities do not change significantly what is compatible with the theoretical results on time deformed models [Ghysels, Gourieroux, Jasiak (1998)]. There is however a substantial decrease in the absolute value of speed adjustment. The volume's $\hat{\alpha}$ in the entire sample decreased by almost 50 %. In the lower panels of Tables 4.1 and 4.2 we show the average variation of estimates within hourly intervals. There is less intraday variability in the return speed adjustments and more in volume's. In volume, we find negative signs of $\hat{\alpha}$ during the first three hours of the working day, while in real time this occurred only in the 12-13 period.

(ii) Correlation analysis and state selection for return series

The resampling at equal intervals modifies significantly the serial correlation in the data. There remains one significant, negative autocorrelation (-0.23) in the quantitative return process, while the persistence range of volume is reduced to two lags. The first order autocorrelation function of returns computed for varying thresholds can exhibit positive values (see Figure 4.5a). However we still find an even function, admitting an optimum around the zero threshold. For this threshold the absolute value of the first order autocorrelation is lower in calendar time than in real time.

The Q^* statistic computed for varying thresholds in Figure 4.5b displays five local maximas, compared to two in real time. They are asymmetric with respect to zero and their number suggest that some positive thresholds may create persistence range up to lag 10 in calendar time.

(iii) Causality analysis and state selection for volume series

Compared to real time data, the quantitative processes of volume and returns are less correlated. There are significant cross-correlations between lagged volume and returns at lags 0, 1 and 8, while only at lag 0 are lagged returns correlated with volume. We repeat the unidirectional one step linear causality test based on regressions of volume (returns) on past volume and returns. Since the relevant coefficients are not significant, the null hypothesis of unidirectional noncausality can not be rejected. There is however strong evidence in favor of instantaneous causality. The regressions for linear causality are [2]:

$$
\begin{aligned}
r_t &= 0.000003 - \underline{0.2372} r_{t-1} - 0.000001 v_{t-1} \\
v_t &= \underline{5.9789} - 4.2148 r_{t-1} + \underline{0.0720} v_{t-1} \\
v_t &= \underline{5.9805} - \underline{476.6650} r_t - 117.2836 r_{t-1} + \underline{0.0716} v_{t-1}
\end{aligned}
$$

Inference on qualitative data with thresholds zero and the sample average for returns and volumes, respectively, is still based on the regression method for unidirectional relationships. We do not reject the null hypothesis of noncausality from returns to volume, but we strongly reject the absence of causality from volume to returns as well as the lack of instantaneous causality. Therefore there exists a nonlinear causality from volume to returns [see Table 4.4].

Figure 4.8 displays the relationship between the causality test statistics and the selected volume thresholds. Amazingly, the F test statistics admit lower values than in real time. Similarly to the real time results, there exists a threshold maximizing the unidirectional causality from volume to returns, and for which the symmetric unidirectional causality is negligeable. Moreover this particular threshold is approximately the same as in real time. We find a substantially larger range of values eliminating causality from returns to volume, including the volume average.

5 Conclusions

This paper examined the dynamics of two-state Markov chains representing quantitative processes of stock returns and volumes. The transitions of the dichotomous qualitative processes were parametrized by the transition probabilities or alternatively by the speed adjustment parameter and the stationary probability. We introduced regression based estimators of the transition parameters and extended the approach to multivariate analysis.

The multiplicity of two-state representation allowed us to focus on qualitative processes featuring some interesting properties. We showed that the range of temporal dependence differs in the quantitative and qualitative data and illustrated the autocorrelation behavior with respect to

[2]statistically significant coefficients are underlined

the state specification. In particular, we found that serially correlated return processes can be transformed into qualitative white noises by selecting an appropriate threshold separating the two states.

The qualitative processes of returns and volumes also display different interactions compared to the quantitative data. In the bivariate setup, we investigated the volume-return relationship and proposed various tests of noncausality hypotheses. Our empirical results indicate that causality directions vary in time and depend on the sampling scheme, such as the real or calendar time scales. We also illustrated the variation of the noncausality test statistic due to adjustments of the state separating threshold. Using the value of test statistic as a measure of strength for a causal relation, we proposed a volume classification based on thresholds maximizing the volume-return causality.

Appendix 1

PROOF OF PROPERTY 1

(i) Consistency

When $T \to \infty$ and $k_T \to 0$, the function $\hat{\rho}_{T,k}(c)$ tends a.s. to $\rho(c)$. When this convergence is uniform, we deduce that $\hat{c}_{T,k}$ exists for T sufficiently large and tends a.s. to c_1.

(ii) Asymptotic Distribution

For T sufficiently large, the estimator satisfies the first order condition:

$$\hat{\rho}_{T,k}(\hat{c}_{T,k}) = 0.$$

Therefore by considering a first order expansion in a neighborhood of the limiting value c_1, we have:

$$\begin{aligned}\sqrt{T}\hat{\rho}_{T,k}(c_1) &\approx \frac{d\hat{\rho}_{T,k}(c_1)}{dc}\sqrt{T}(\hat{c}_{T,k} - c_1) + o_p(1)\\ &\approx \frac{d\rho(c_1)}{dc}\sqrt{T}(\hat{c}_{T,k} - c_1) + o_p(1),\end{aligned}$$

or equivalently:

$$\sqrt{T}(\hat{c}_{T,k} - c_1) \approx \left[\frac{d\rho(c_1)}{dc}\right]^{-1}\sqrt{T}\hat{\rho}_{T,k}(c_1) + o_p(1).$$

If k_T tends to zero sufficiently fast we get:

$$\sqrt{T}(\hat{c}_{T,k} - c_1) = \left[\frac{d\rho(c_1)}{dc}\right]^{-1}\sqrt{T}\hat{\rho}_{T}(c_1) + o_p(1).$$

For the value c_1, we have $\rho(c_1) = 0$ and it is known that: $\sqrt{T}\hat{\rho}_T(c_1) \xrightarrow{d} N(0,1)$, [see Fuller (1976)]. Moreover, using conventional notation, we have:

$$\frac{d}{dc}[\rho(c_1)] = \frac{d}{dc}\left[\frac{\gamma_1(c_1)}{\gamma_0(c_1)}\right] = \frac{1}{\gamma_0(c_1)}\frac{d\gamma_1(c_1)}{dc},$$

$$\begin{aligned}\left[\frac{d\gamma_1(c_1)}{dc}\right]_{c=c_1} &= \left[\frac{d}{dc}E(1_{Y_t>c}1_{Y_{t-1}>c})\right]_{c=c_1} - \left(\frac{d}{dc}\left[E(1_{Y_t>c})\right]^2\right)_{c=c_1}\\ &= \left[\frac{d}{dc}\int_c^\infty\int_c^\infty f(y_t, y_{t-1})dy_t dy_{t-1}\right]_{c=c_1} + 2P[Y_t > c_1]f(c_1)\end{aligned}$$

$$
\begin{aligned}
&= -\int_{c_1}^{\infty} f(c_1, y_{t-1}) dy_{t-1} - \int_{c_1}^{\infty} f(y_t, c_1) dy_t + 2f(c_1)P[Y_t > c_1] \\
&= -f(c_1)[P[Y_{t-1} > c_1 | Y_t = c_1] + P[Y_t > c_1 | Y_{t-1} = c_1] - 2P[Y_t > c_1]].
\end{aligned}
$$

FIG 4.1: Alcatel returns, 07-08.1996

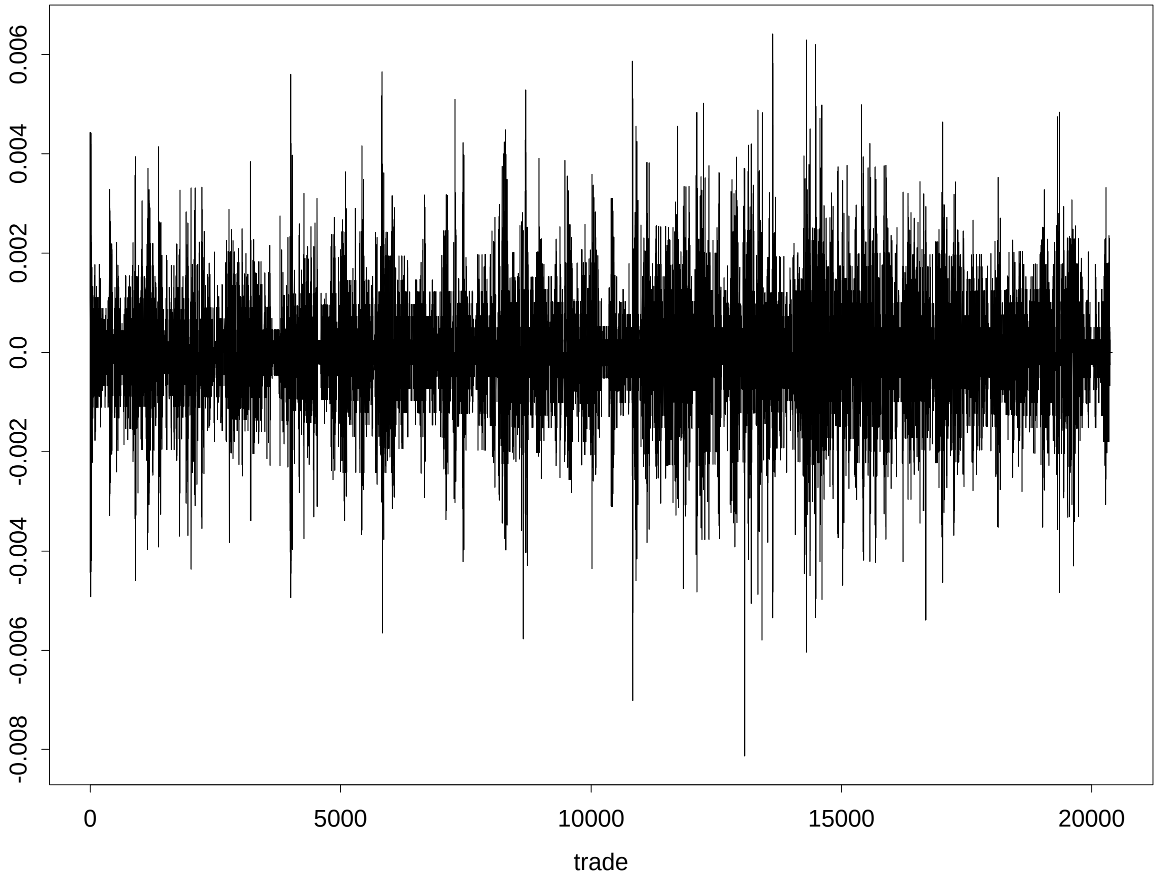

FIG 4.2a: RETURNS _______ m

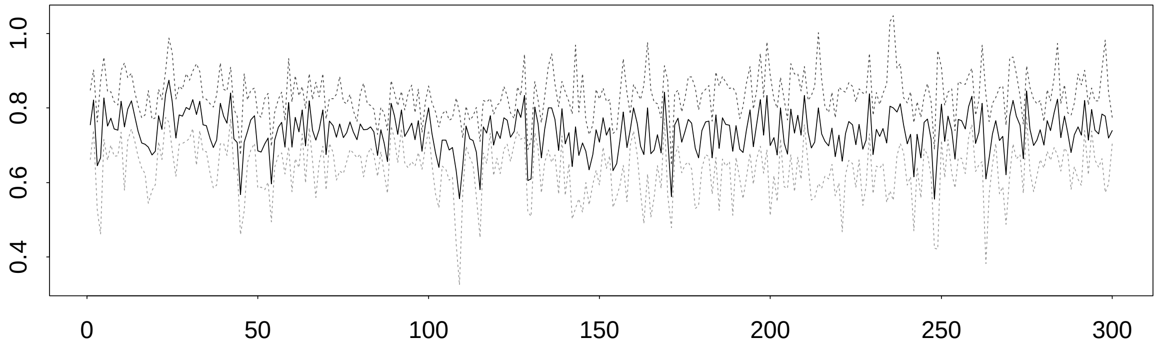

FIG 4.2b: VOLUME _______ m

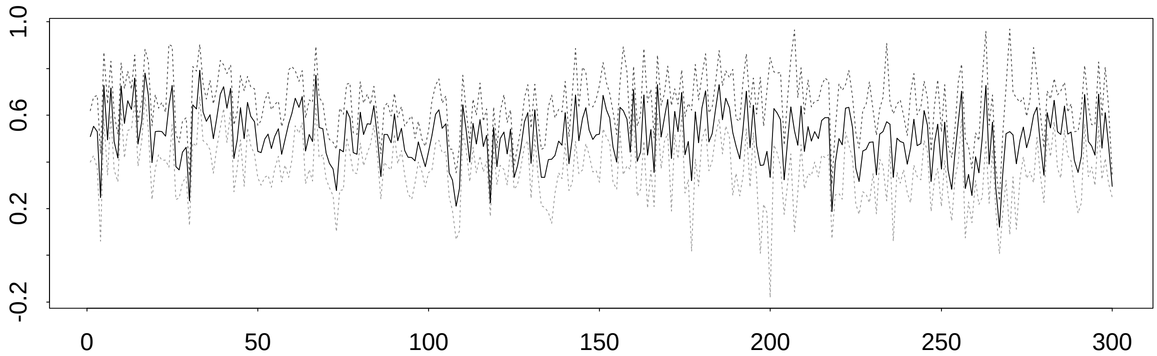

FIG 4.3a: RETURNS _______ alpha

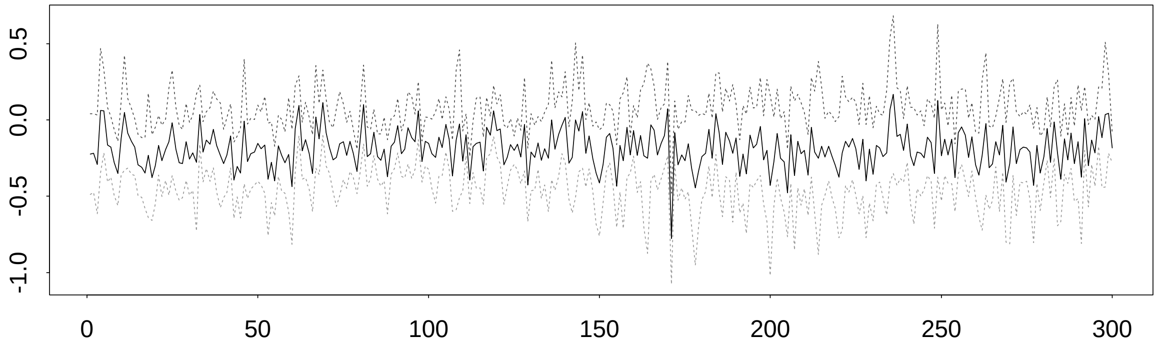

FIG 4.3b: VOLUME _______ alpha

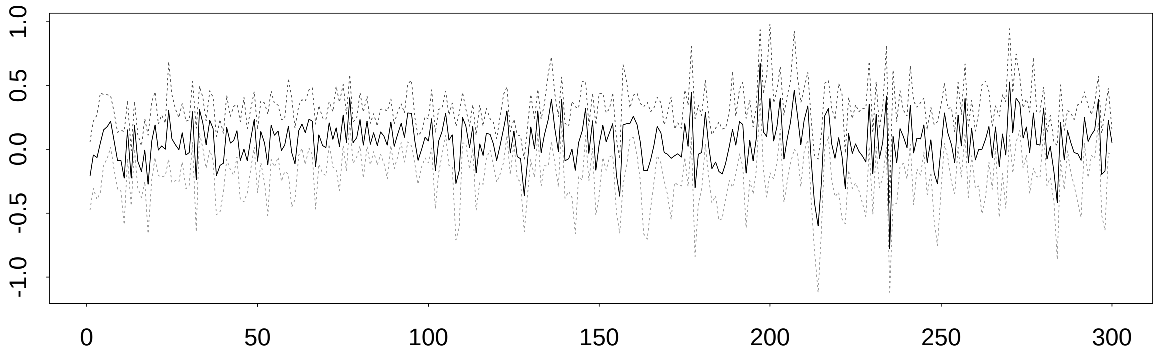

FIG 4.4 : REAL TIME: threshold effect in return persistence

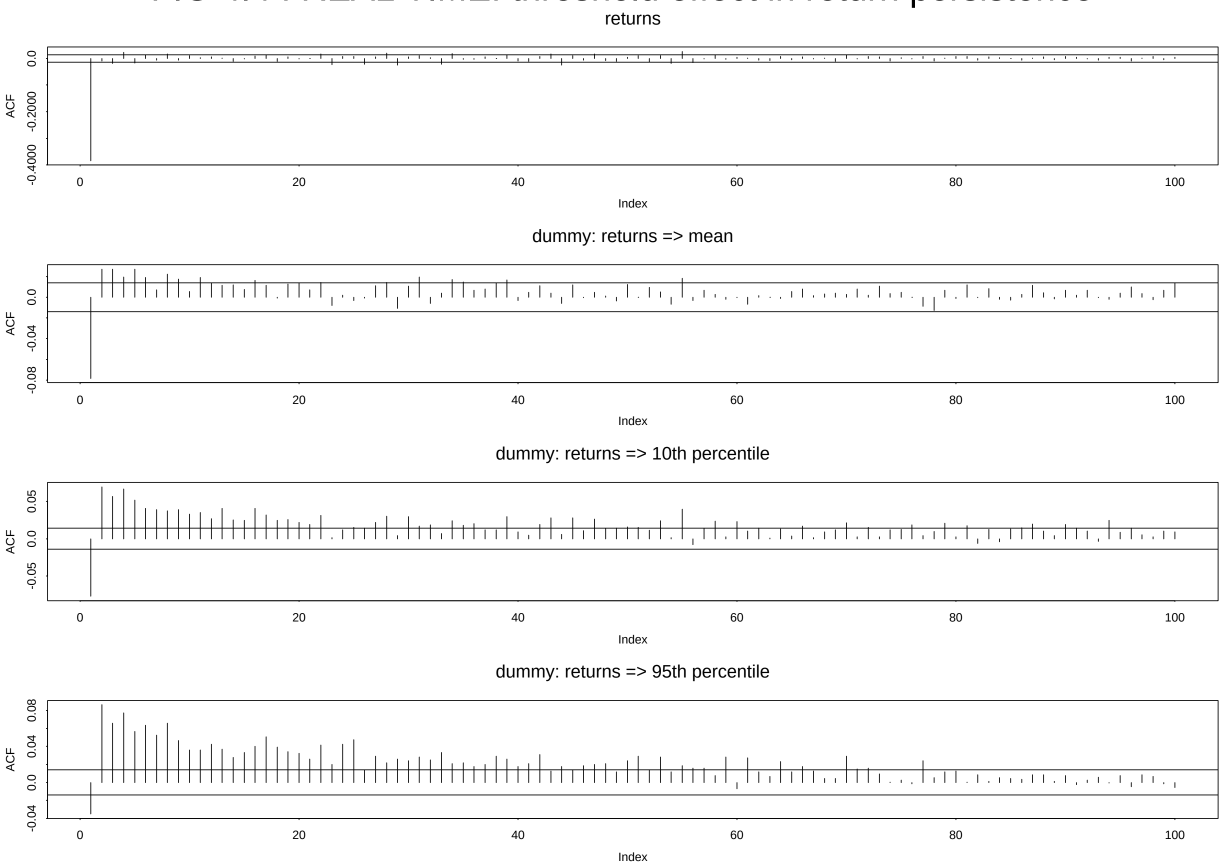

FIG 4.5a Correlation at lag 1 for varying thresholds

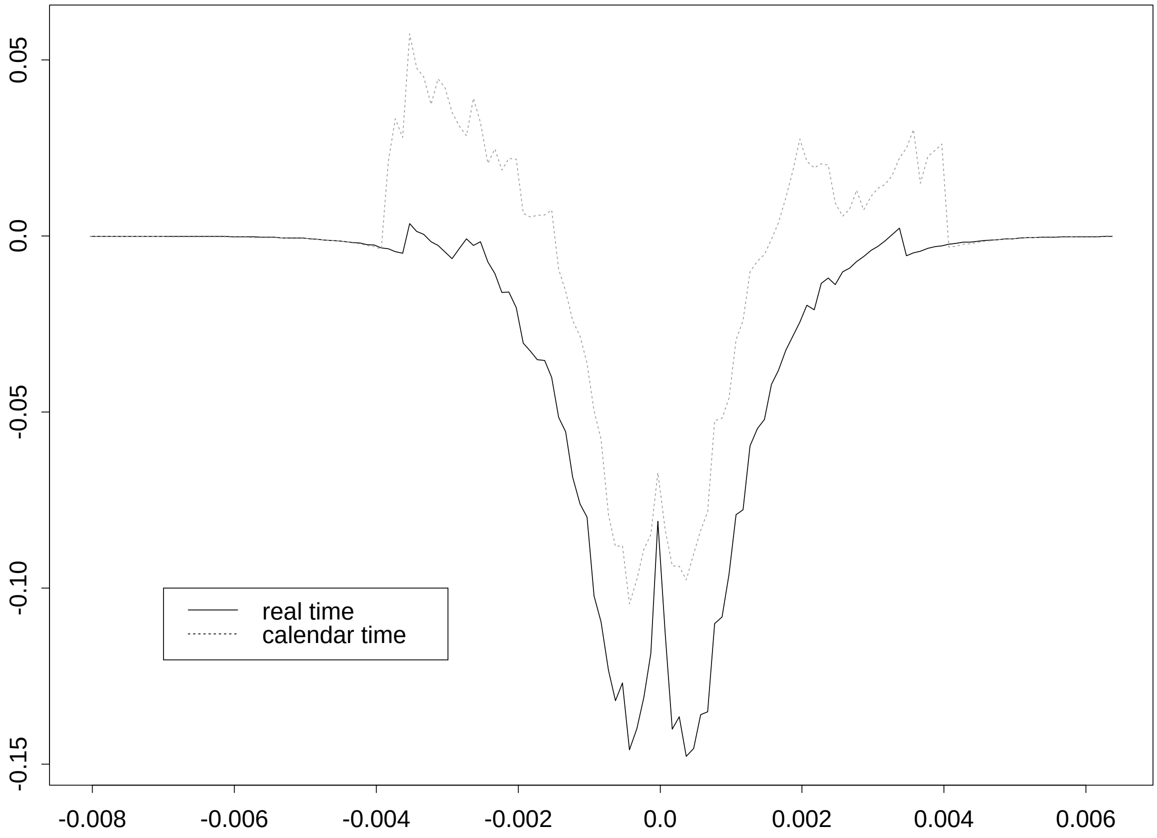

FIG 4.5b Q*-statistic for varying thresholds
lags 2 to 10

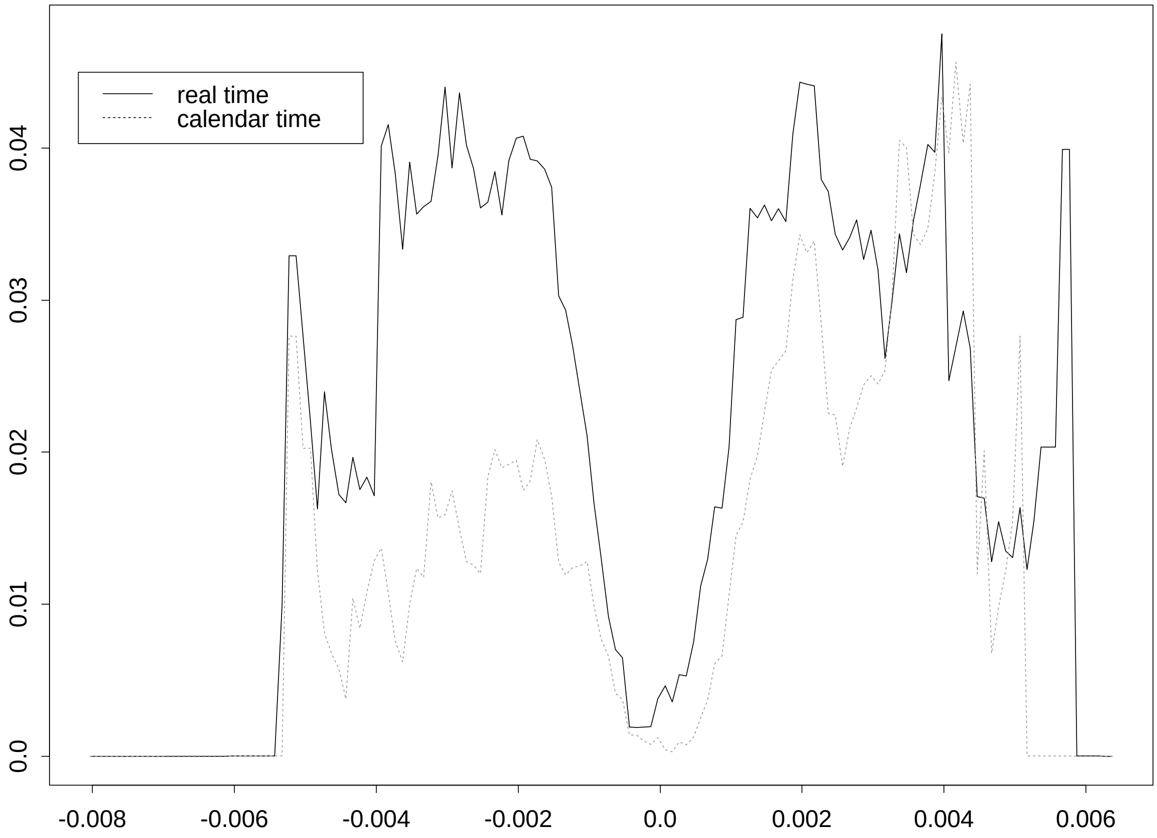

FIG 4.6 REAL TIME: daily dynamics in causality
P-value

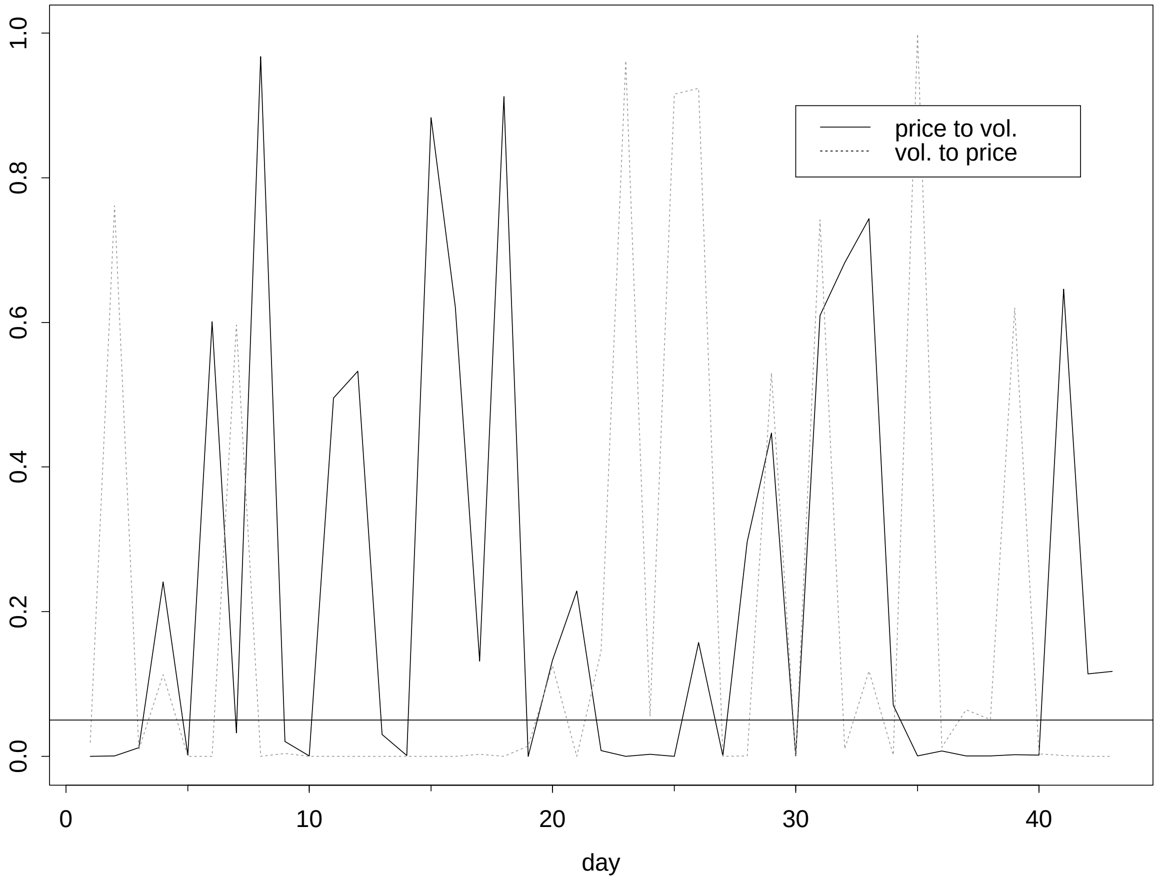

FIG 4.7 REAL TIME: threshold effect in causality
F test

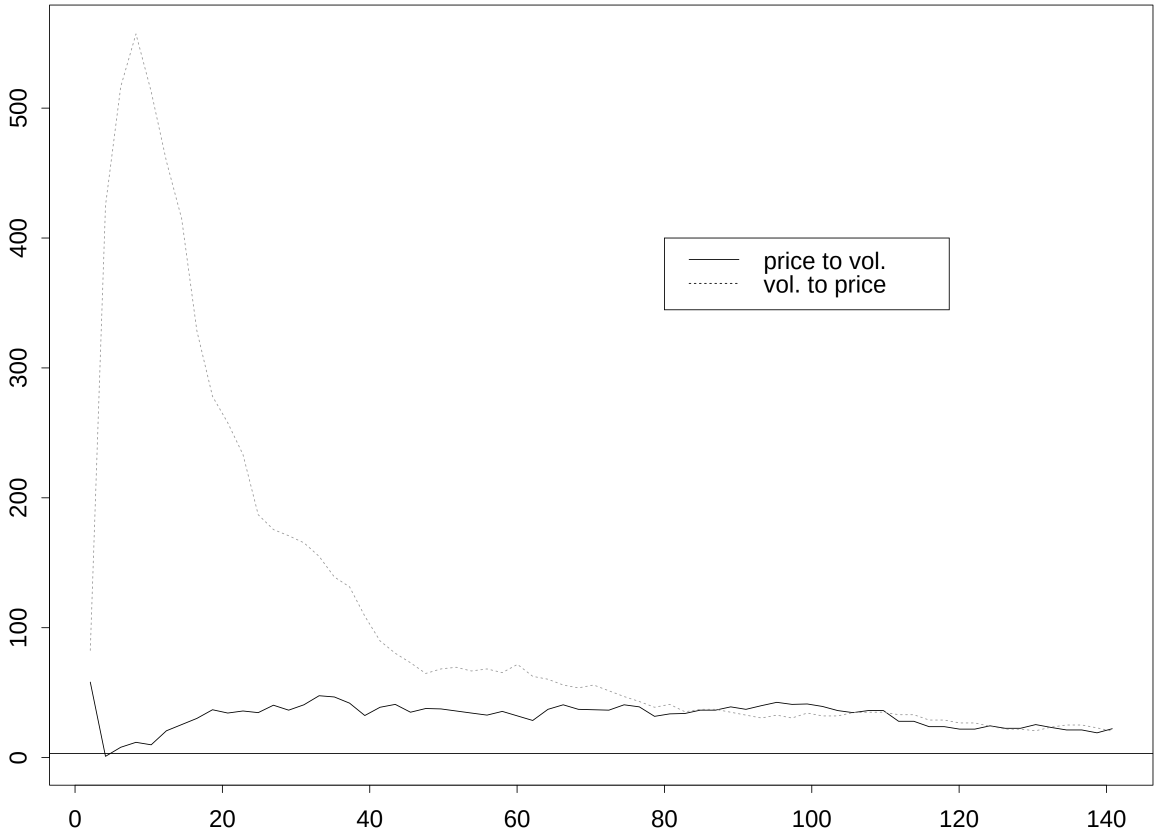

FIG 4.8 CALENDAR TIME: threshold effect in causality
F test

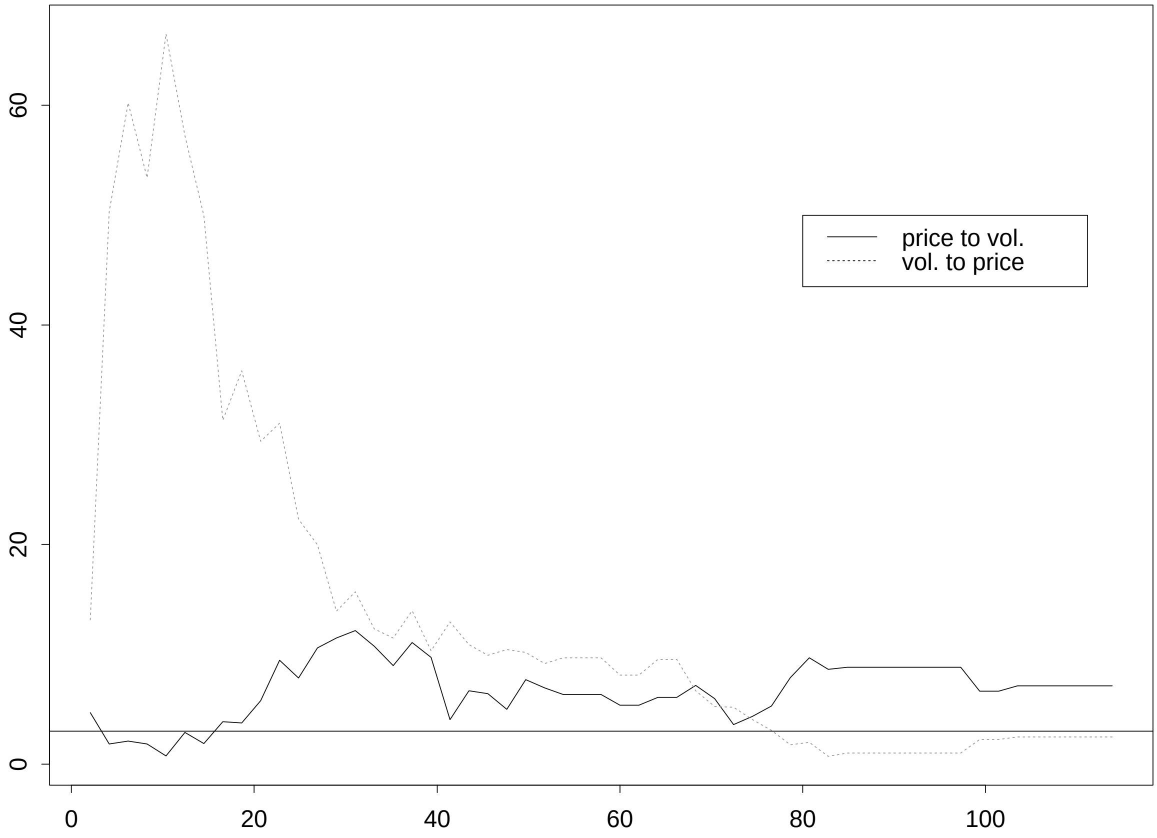

Table 4.1

Hourly averages of m and α coefficients

ALCATEL RETURNS ($Z(t) = 1$ if $\mathrm{ret}(t) \geq 0$)

REAL TIME

hour	mean m	S.D.	mean alpha	S.D.
10–11	0.6785	0.0679	–0.1104	0.1228
11–12	0.6843	0.0647	–0.1725	0.1252
12–13	0.6807	0.0834	–0.1351	0.1418
13–14	0.6464	0.0979	–0.1107	0.2447
14–15	0.6755	0.0834	–0.0706	0.1601
15–16	0.6665	0.0652	–0.0938	0.1358
16–17	0.7026	0.0570	–0.0917	0.0955

CALENDAR TIME

hour	mean m	S.D.	mean alpha	S.D.
10–11	0.6864	0.0986	–0.0785	0.1759
11–12	0.6827	0.0745	–0.1195	0.1481
12–13	0.6690	0.1117	–0.1180	0.2212
13–14	0.6527	0.1090	–0.1230	0.2809
14–15	0.6804	0.1002	–0.1050	0.1618
15–16	0.6757	0.0764	–0.0995	0.1573
16–17	0.7010	0.0723	–0.1047	0.1497

Table 4.2

Hourly averages of m and α coefficients

ALCATEL VOLUME ($Z(t) = 1$ if $\mathrm{vol}(t) \geq \mathrm{mean(vol)}$)

hour	mean m	S.D.	mean alpha	S.D.
REAL TIME				
10–11	0.2517	0.0743	0.0548	0.1492
11–12	0.2426	0.0777	0.0295	0.1494
12–13	0.2843	0.1101	–0.0386	0.1732
13–14	0.3299	0.1366	0.0360	0.1969
14–15	0.3138	0.1029	0.0902	0.1860
15–16	0.3149	0.0785	0.0447	0.1562
16–17	0.3203	0.0999	0.0445	0.1071
CALENDAR TIME				
10–11	0.2499	0.0863	–0.0357	0.1712
11–12	0.2304	0.0791	–0.0027	0.1717
12–13	0.2829	0.1133	–0.0795	0.2169
13–14	0.3297	0.1310	0.0158	0.2475
14–15	0.2795	0.1083	0.0324	0.1930
15–16	0.2910	0.0923	0.0142	0.1714
16–17	0.3384	0.1056	–0.0176	0.1502

Table 4.3: REAL TIME: CAUSALITY

UNIDIRECTIONAL					
PARAMETER	EST	S.E.	PARAMETER	EST	S.E.
$\beta_{1.}$	0.2739	0.0070	$\beta_{.1}$	0.8372	0.0071
$\alpha_{1.\|1.}$	0.1092	0.0117	$\alpha_{.1\|1.}$	–0.2775	0.0118
$\alpha_{1.\|.1}$	–0.0182	0.0083	$\alpha_{.1\|.1}$	–0.1795	0.0084
$\alpha_{1.\|11}$	–0.0094	0.0146	$\alpha_{.1\|11}$	0.2814	0.0147
F-test: 10.10, p-value = 0.000			F-test: 548.04, p-value = 0.000		
INSTANTANEOUS					
PARAMETER	EST	S.E.	PARAMETER	EST	S.E.
γ_1	0.5297	0.0172	$\delta_{11.}$	0.1439	0.0265
$\delta_{1..}$	–0.3055	0.0188	$\delta_{1.1}$	0.2234	0.0210
$\delta_{.1.}$	–0.0560	0.0222	$\delta_{.11}$	0.0676	0.0266
$\delta_{..1}$	–0.2200	0.0188	δ_{111}	–0.0100	0.0322
F-test: 426.47, p-value = 0.0000					

Table 4.4: CALENDAR TIME: CAUSALITY

UNIDIRECTIONAL					
PARAMETER	EST	S.E.	PARAMETER	EST	S.E.
$\beta_{1.}$	0.2733	0.0099	$\beta_{.1}$	0.7755	0.0101
$\alpha_{1.\vert 1.}$	0.0573	0.0171	$\alpha_{.1\vert 1.}$	–0.1371	0.0175
$\alpha_{1.\vert .1}$	–0.0056	0.0117	$\alpha_{.1\vert .1}$	–0.1066	0.0121
$\alpha_{1.\vert 11}$	–0.0150	0.0212	$\alpha_{.1\vert 11}$	0.1126	0.0218
F-test: 1.59, p-value = 0.2024			F-test: 64.35, p-value = 0.000		
INSTANTANEOUS					
PARAMETER	EST	S.E.	PARAMETER	EST	S.E.
γ_1	0.3995	0.0208	$\delta_{11.}$	0.0350	0.0373
$\delta_{1..}$	–0.1626	0.0236	$\delta_{1.1}$	0.0834	0.0272
$\delta_{.1.}$	0.0126	0.0310	$\delta_{.11}$	–0.0551	0.0375
$\delta_{..1}$	–0.0787	0.0235	δ_{111}	0.0935	0.0455
F-test: 106.27, p-value = 0.0000					

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